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6.002 Recitation Notes – Spring 2020

Dependent Sources and Examples

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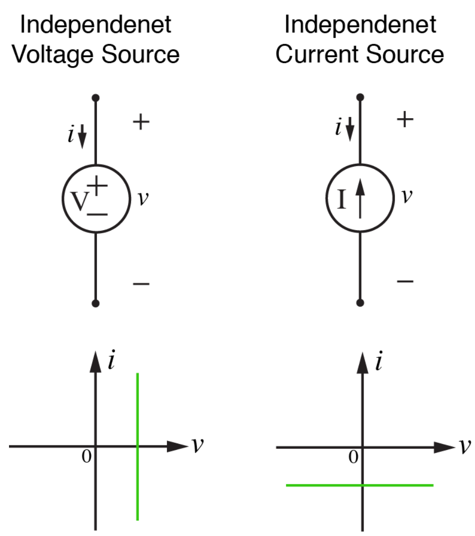
Reference: "Foundations of Analog and Digital Electronics Circuits", Chapters 2.6, Chapter 3

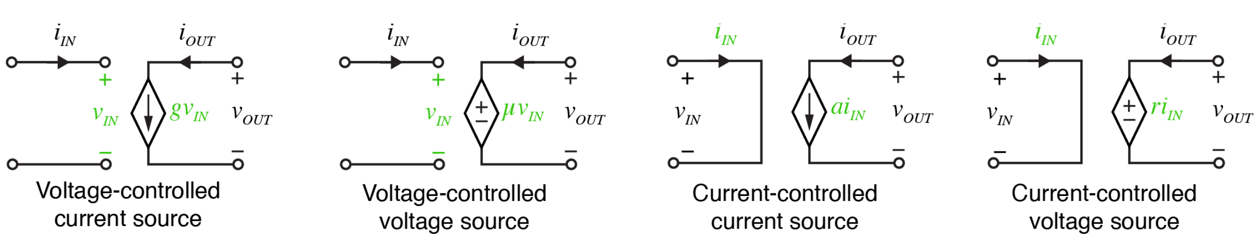
Independent Sources vs. Dependent Sources

Independent voltage and current sources are idealized circuit elements which set the branch voltage or current, respectively, at a specific constant value, independent of the circuit operation. The symbols and current-voltage (*i*-*v*) characteristics of an independent voltage and current source are shown below.



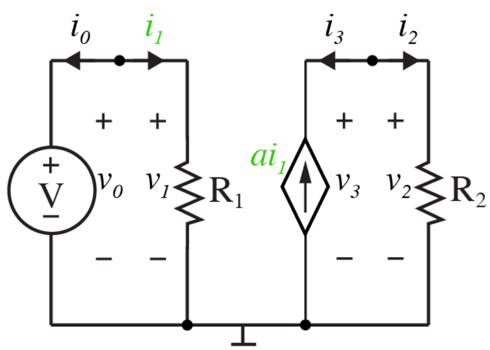
Dependent voltage and current sources are sources whose values depend on (is controlled by) a voltage or current elsewhere in the circuit. These sources are often used for modeling circuit components with more than two terminals, for example amplifiers.

There are 4 types of dependent sources: voltage-controlled current source (VCCS), voltage-controlled voltage source (VCVS), current-controlled current source (CCCS), current-controlled voltage source (CCVS). These are summarized below.



Circuits including dependent sources can be analyzed using the Nodal analysis, Intuitive, Thévenin/Norton, and Superposition methods. Here we will go through some example circuits with dependent sources.

Example 1 – Current-Controlled Current Source
Determine the branch variables associated with R_2 (that is i_2 and v_2). Let's use an intuitive approach to analyze this circuit.



$$v_2 = i_2 R_2$$
$$i_2 = \alpha i_1$$

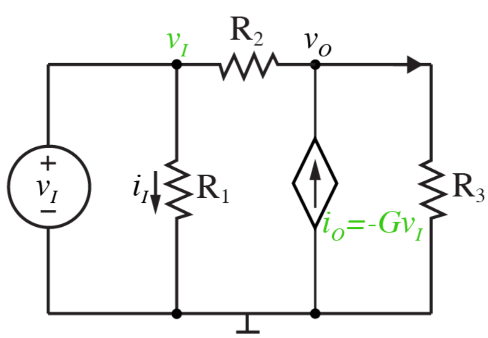
By finding i_1 we can determine v_2 and i_2.

$$i_1 = \frac{v_1}{R_1} = \frac{V}{R_1}$$

Substitute i_1 back into the equation for i_2 and then i_2 back into the equation for v_2.

$$i_2 = \alpha i_1 = \alpha \frac{V}{R_1}$$
$$v_2 = i_2 R_2 = \alpha \frac{V R_2}{R_1}$$

Example 2 – Voltage-Controlled Current Source
Determine v_O as a function of v_I.



Let's use nodal analysis to solve for v_O. As a reminder, the 5 steps involved in nodal analysis are summarized below.

1. Select a reference node (ground).
2. Label the potentials of the remaining nodes with respect to the ground.
3. Write KCL for each node that has an unknown node voltage.
4. Solve the resulting equations.
5. Back-solve for the branch voltages and currents.

Steps 1 and 2 are already completed.
Step 3 and 4 – Write KCL at the upper right node and solve for v_O.

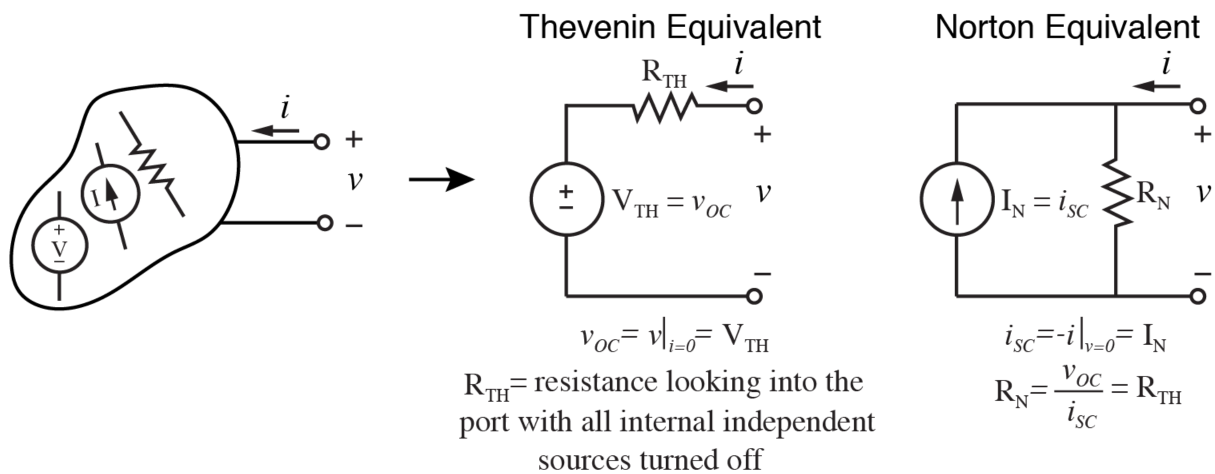
$$\frac{v_I - v_O}{R_2} + (-G v_I) = \frac{v_O}{R_3}$$
$$v_O = \frac{(1 - G R_2) R_3}{R_2 + R_3} v_I$$

Example 3 – Thévenin and Norton Equivalent Network with Dependent Sources

Circuits involving dependent sources can be analyzed using the Thévenin and Norton methods only if: 1) the dependent source is linear, and 2) both parts of the dependent source are inside the circuit being modeled.

Why are the Thévenin and Norton methods helpful?

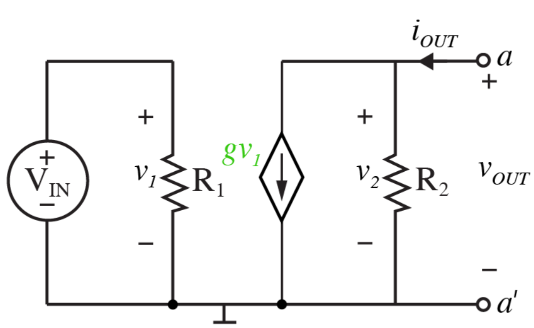
In a linear system, they allow us to represent any collection of circuit elements at a pair of terminals by one voltage source and one resistor (Thévenin equivalent), or by one current source and one resistor (Norton equivalent).



When using the Thévenin and Norton methods, there are two main steps to follow:

1. Find V_{TH} or I_N
2. Find $R_{TH} = R_N$

Find the Thévenin and Norton equivalent networks for the following circuit.



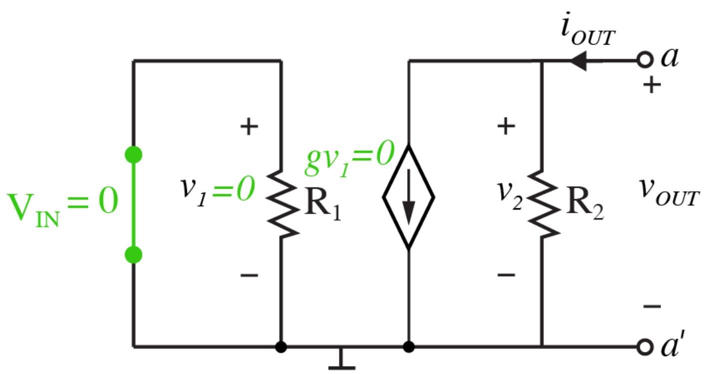
Note that this circuit has a voltage-controlled current source.

To find the **Thévenin equivalent** for this circuit, we need to find V_{TH} and R_{TH}.

1. Find V_{TH}.
   To find V_{TH}, determine v_{out} when $i_{out} = 0$.

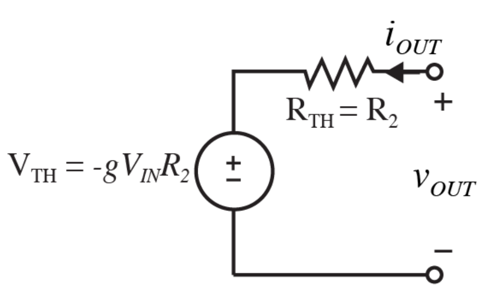
   $V_{TH} = v_{OC} = -gv_1R_2$
   $v_1 = V_{IN}$
   $V_{TH} = v_{OC} = -gV_{IN}R_2$

2. Find R_{TH}.
   To find R_{TH}, find the resistance looking into the aa' port with the internal independent source turned off. Here, we have a voltage source and turning off a voltage source means having it shorted. Note that you should not be turning off the dependent sources.



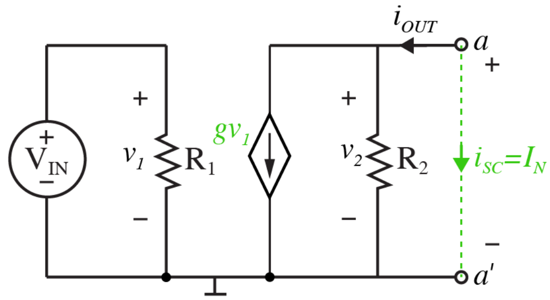
Turn off the independent voltage source such that $V_{IN} = v_1 = 0$.
As a result, the dependent current source is also $gv_1 = 0$.
The resistance looking into the aa' port gives us the $R_{TH} = R_2$

The Thévenin equivalent form for the circuit in this example based on V_{TH} and R_{TH} is shown below.



To find the **Norton equivalent** for this circuit, we need to find I_N and R_N.

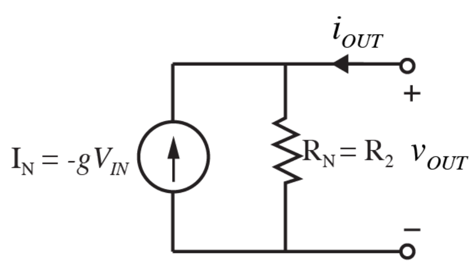
1. Find I_N.
   To find I_N, determine i_{out} when $v_{out} = 0$.



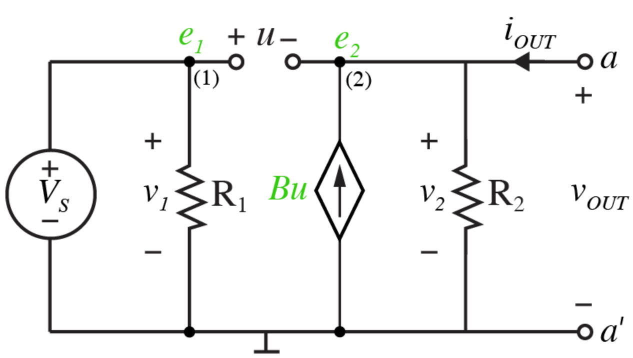
$I_N = i_{SC} = -gv_1 = -gV_{IN}$

2. Find R_N.
   $R_N = R_{TH} = R_2$

The Norton equivalent form for the circuit in this example based on I_N and R_N is shown below.



Example 4 – Thévenin and Norton Equivalent Network with Dependent Sources
Consider the circuit below and find its Thévenin equivalent network.

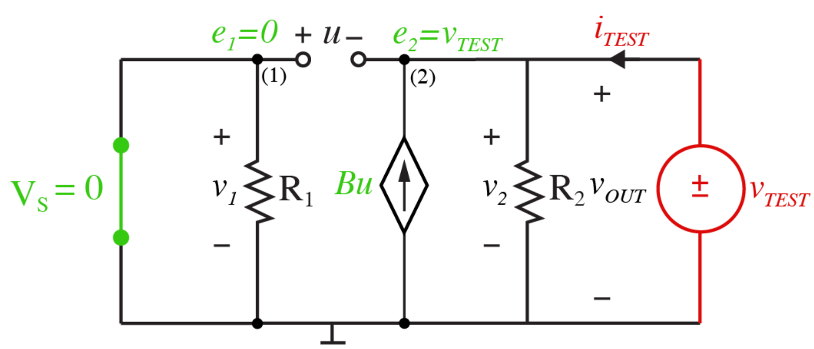


To find the **Thévenin equivalent** for this circuit, we need to find V_{TH} and R_{TH}.

1. Find V_{TH}.
   To find V_{TH}, determine v_{out} when $i_{out} = 0$.

$$V_{TH} = v_{OUT} = e_2 = BuR_2$$
$$u = e_1 - e_2 = V_S - BuR_2$$
$$u = \frac{V_S}{1 + BR_2}$$
$$V_{TH} = v_{OUT} = e_2 = \frac{BR_2}{1 + BR_2} V_S$$

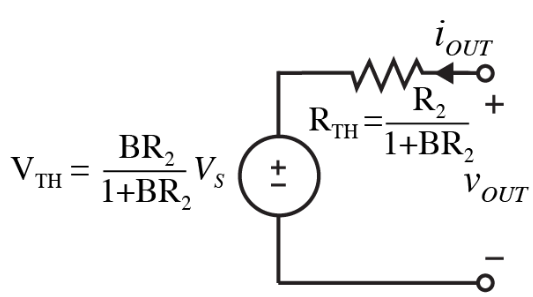
2. Find R_{TH}.
   To find R_{TH}, find the resistance looking into the aa' port with the internal independent source turned off. Here, to help us with calculating R_{TH} we are going to use a test voltage source.



$$R_{TH} = \frac{v_{TEST}}{i_{TEST}}$$

Write a KCL at node 2:
$$Bu + i_{TEST} - \frac{v_{TEST}}{R_2} = 0$$
Note that $u = e_1 - e_2 = -v_{TEST}$

$$-Bv_{TEST} + i_{TEST} - \frac{v_{TEST}}{R_2} = 0$$
$$v_{TEST}\left(B + \frac{1}{R_2}\right) = i_{TEST}$$
$$R_{TH} = \frac{v_{TEST}}{i_{TEST}} = \frac{1}{B + \frac{1}{R_2}} = \frac{R_2}{BR_2 + 1}$$



Note that another approach to finding R_{TH} is to find the i_{sc} and consider that $R_{TH}=R_N=v_{oc}/i_{sc}$.