6.002 Recitation Notes – Spring 2020

MOSFET and Large-Signal MOSFET Amplifier

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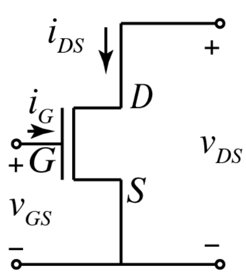
Reference: “Foundations of Analog and Digital Electronics Circuits”, Chapters 6, 7, 8

Review of MOSFET

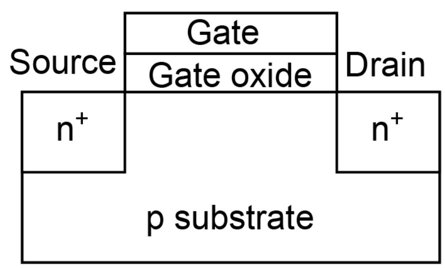
MOSFET is a three-terminal device with a control terminal (Gate, G), an input terminal (Drain, D) and an output terminal (Source, S), shown by the symbol below. The gate voltage (v_{GS}) helps control the current flowing from the drain to the source (i_{DS}).

The schematic below shows a basic physical structure of an n-channel MOSFET. It consists of a p-type silicon substrate with regions that are heavily n-doped to define the source and the drain. The gate is separated from the substrate by a thin oxide layer. In this design, if $v_{GS} = 0$ and $v_{DS} > 0$, no current will flow from the drain to the source ($i_{DS} = 0$). This is because the n-type source and drain are separated by the p-type region. However, when a positive voltage is applied at the gate ($v_{GS} > 0$), negative charges are attracted to the surface from the nearby negatively charged source and drain regions and positive charges are repelled from the surface. As v_{GS} increases, more negative charges are attracted to the surface until they form an n-type conducting channel that connects the source and the drain when v_{GS} exceeds a threshold voltage (v_T). At this point, a current begins to flow between the drain and the source when $v_{DS} > 0$.

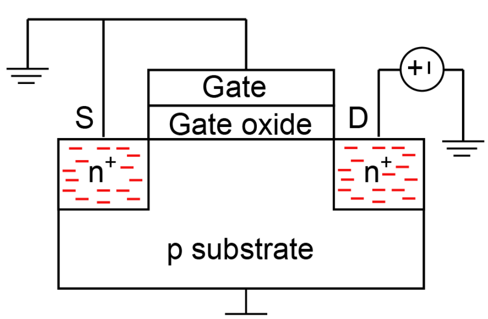
MOSFET Symbol



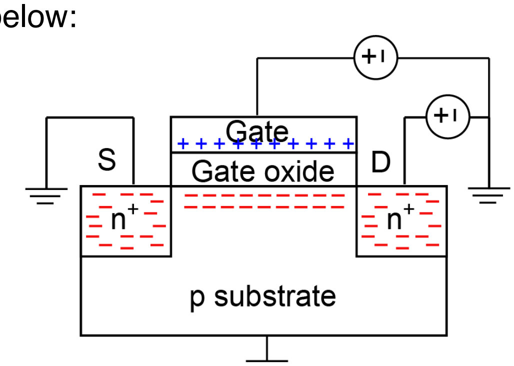
Basic MOSFET Physical Structure



The basic operation of the MOSFET is shown below:

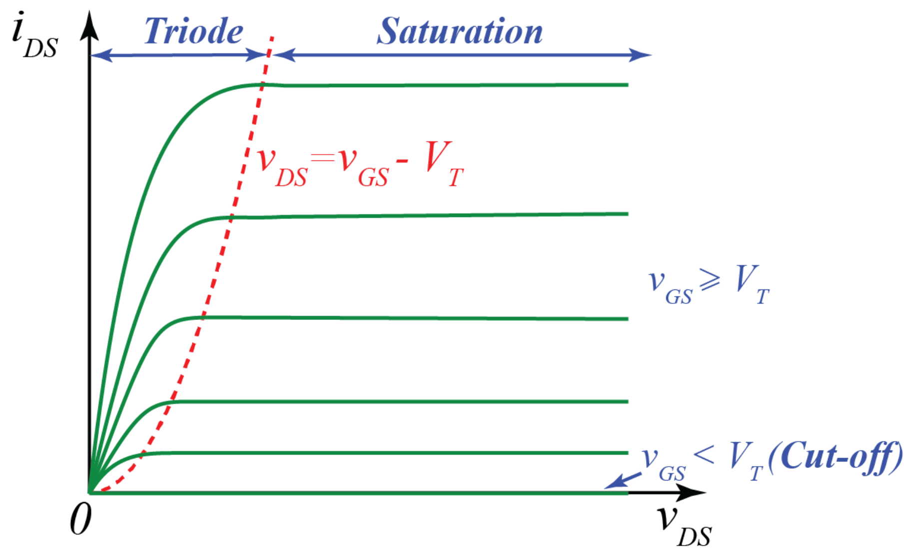


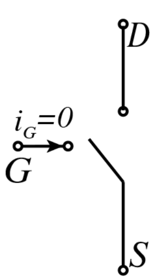
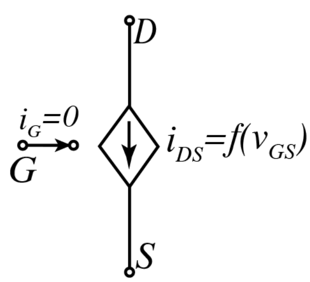
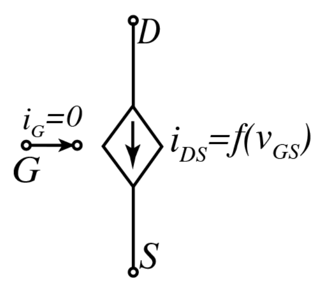
$v_{GS} = 0, v_{DS} > 0 \Rightarrow i_{DS} = 0$



$v_{GS} \geq V_T, v_{DS} > 0 \Rightarrow i_{DS} > 0$

MOSFET's three regions of operation and their corresponding large signal models are summarized below.



Cut-off	**Saturation**	**Triode (Linear)**
$v_{GS} < V_T$	$v_{GS} \geq V_T$ $v_{DS} \geq v_{GS} - V_T$	$v_{GS} \geq V_T$ $v_{DS} < v_{GS} - V_T$
$i_{DS} = 0$	$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$	$i_{DS} = K\left[(v_{GS} - V_T)v_{DS} - \frac{v_{DS}^2}{2}\right]$
Open State	Closed State	Closed State
		

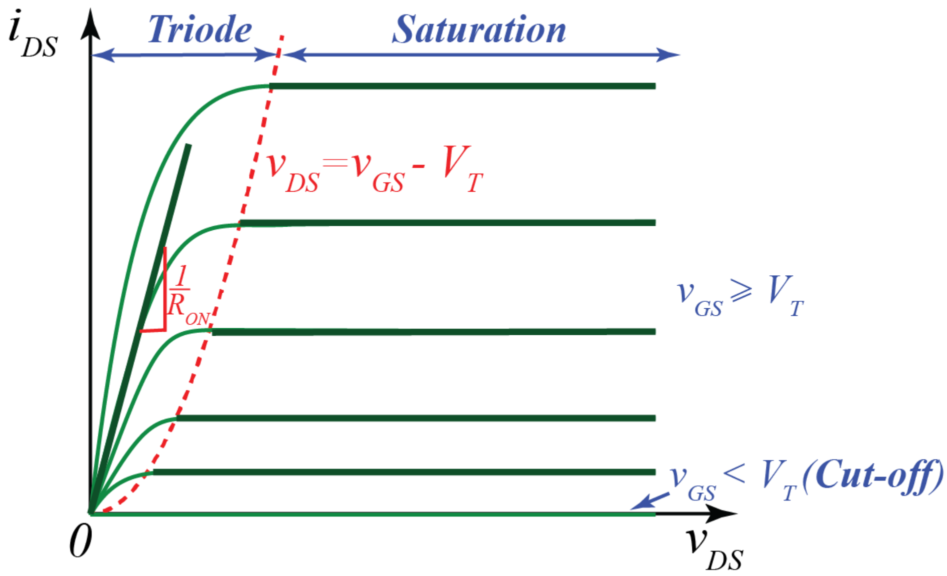
Note that for the Triode region since $v_{DS} < v_{GS} - V_T$ we can simplify i_{DS} such that it can be approximated having a linear dependence on v_{DS}.

$$i_{DS} = K(v_{GS} - V_T)v_{DS} - \frac{Kv_{DS}^2}{2}$$

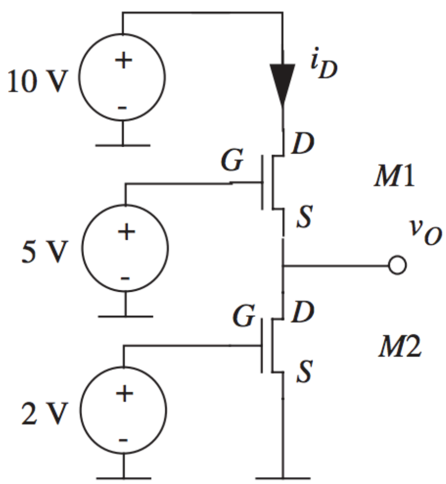
Since $v_{DS} < v_{GS} - V_T$

$$i_{DS} \approx K(v_{GS} - V_T)v_{DS}$$

$$\frac{v_{DS}}{i_{DS}} \approx \frac{1}{K(v_{GS} - V_T)} \approx R_{ON}$$



Let's consider a simple example of a circuit with two MOSFETs. Assuming operation in the saturation region, what is v_0? Assume the two MOSFETs are identical, K = 4 mA/V^2 and V_T = 1V.

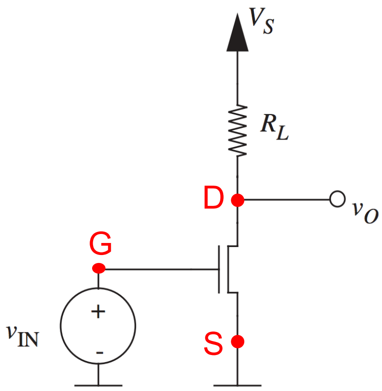


MOSFETs are identical, i_{DS} values for both MOSFETs are the same, v_{GS} for the two MOSFETs must be equal. Note that in the saturation region, i_{DS} is not dependent on v_{DS}.

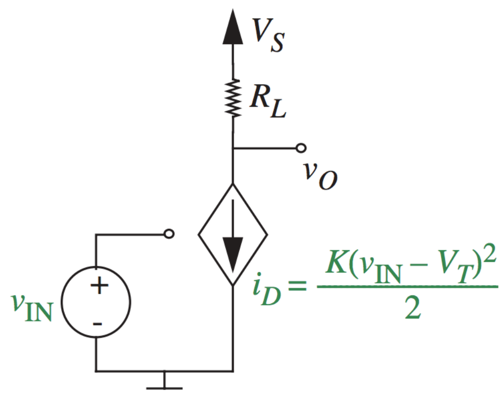
$$5 - v_0 = 2$$
$$v_0 = 3\ V$$

The MOSFET Amplifier

In a MOSFET amplifier, the input and output voltages are selected so that the MOSFET operates in its saturation region. A MOSFET amplifier circuit and its equivalent model in the saturation region are shown below.



MOSFET amplifier



Saturation region model

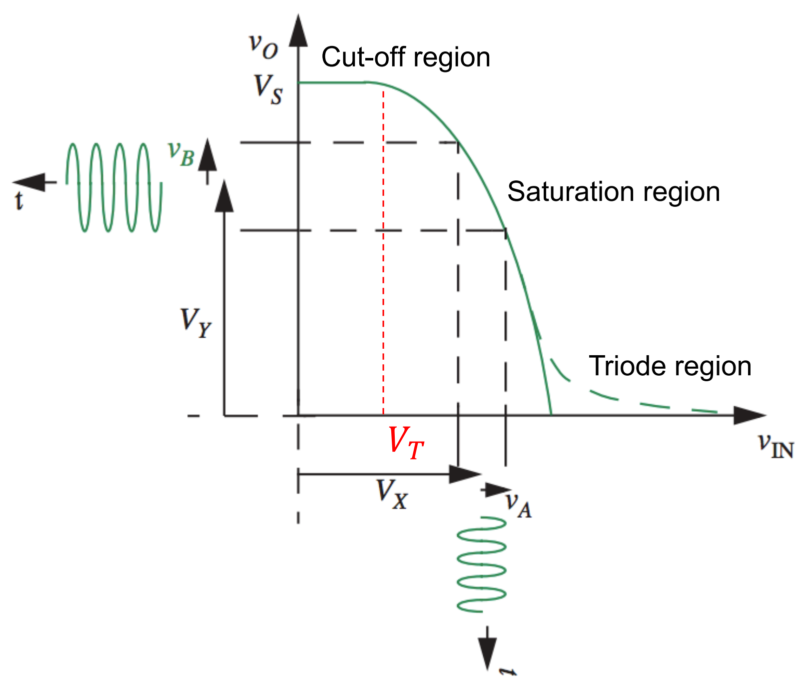
$$v_{GS} = v_{IN}$$
$$v_{DS} = v_O$$

In saturation, $v_{IN} \geq V_T$ and $v_O \geq v_{IN} - V_T$

$$i_{DS} = \frac{K}{2}(v_{IN} - V_T)^2$$

The transfer function of the amplifier in the saturation region is shown below. The transfer function shows the relation between v_O and v_{IN}.

$$i_D = \frac{V_S - v_O}{R_L}$$
$$v_O = V_S - R_L i_D$$
$$v_O = V_S - R_L \frac{K}{2}(v_{IN} - V_T)^2$$



For amplification to be effective, the circuit needs to operate in the saturation region. To ensure the operation is maintained in this region, we can boost the signal that needs to be amplified with a DC offset so that the amplifier operates in its saturation region ($v_{IN} = V_X + v_A$).

Another point to consider is that the slope of the transfer curve at different points in the saturation region is different suggesting that the gain is nonlinear. However, the gain can be approximated to be linear if the circuit is operating with a very small signal about an operating point. We will learn about the small signal response of the MOSFET in detail in the future lectures, for now we will focus on the large signal analysis.

Large-signal analysis of an amplifier attempts to answer the following questions:

1. What is the relationship between the amplifier output v_O and its input v_{IN} in the saturation region? On the previous page we calculated this analytically. Other techniques that we have learned for analyzing nonlinear devices such as the graphical analysis can also be used.

2. What is the range of valid input and output values for the amplifier? That is, what range of input voltages (and the resulting range of output voltages) will have the MOSFET in the circuit operate in the saturation region.

For the MOSFET to be in the saturation region:
$v_{DS} \geq v_{GS} - V_T$ and $v_{GS} \geq V_T$

Lower limit of the acceptable range (shows transition from cut-off to saturation region):
Note that $v_{DS} = v_O$ and $v_{GS} = v_{IN}$,

$v_{IN} = V_T$
$v_O = V_S - R_L \frac{K}{2}(v_{IN} - V_T)^2$ substitute $v_{IN} = V_T$
$v_O = V_S$

Higher limit of the acceptable range (shows transition from saturation to triode region)
$v_{DS} = v_{GS} - V_T$, noting that $v_{DS} = v_O$ then $v_O = v_{IN} - V_T$

$$v_O = V_S - R_L \frac{K}{2}(v_{IN} - V_T)^2$$

$$v_{IN} - V_T = V_S - R_L \frac{K}{2}(v_{IN} - V_T)^2$$

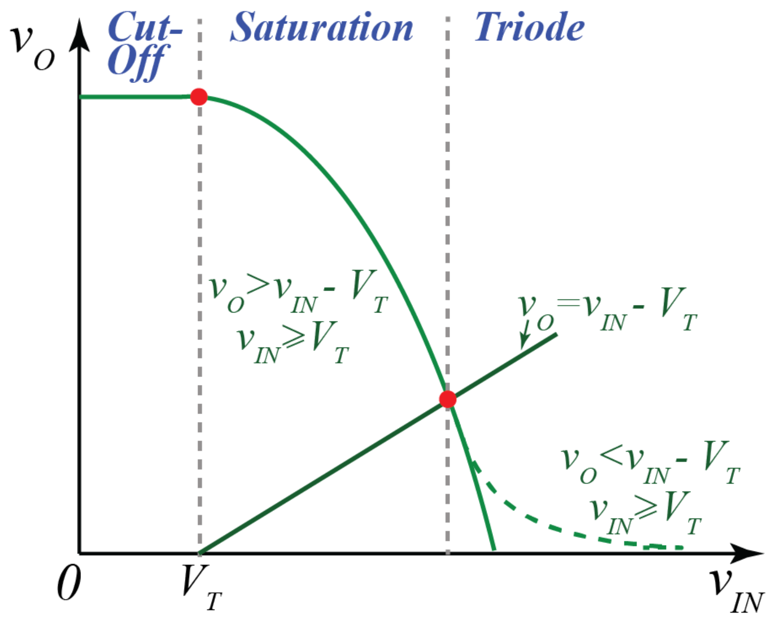
$$R_L \frac{K}{2}(v_{IN} - V_T)^2 + (v_{IN} - V_T) - V_S = 0$$

$$v_{IN} - V_T = \frac{-1 + \sqrt{1 + 2V_S R_L K}}{R_L K}$$

$$v_{IN} = \frac{-1 + \sqrt{1 + 2V_S R_L K}}{R_L K} + V_T$$

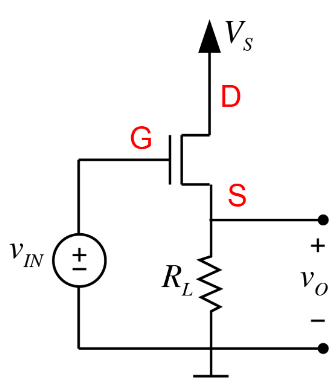
$$v_O = v_{IN} - V_T = \frac{-1 + \sqrt{1 + 2V_S R_L K}}{R_L K}$$

Note that the acceptable range could have also been determined graphically. This is schematically shown below.

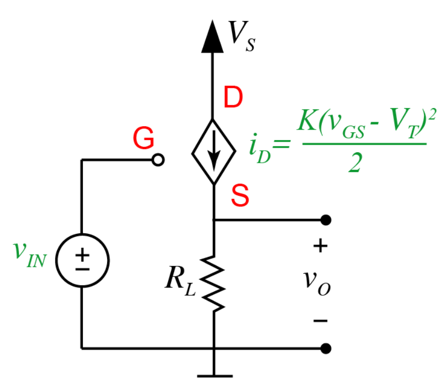


Example: MOSFET Source Follower Circuit

Below is an example of a source follower circuit and the corresponding large signal model in the saturation region.



MOSFET Source Follower



Saturation Region Model

In saturation, $v_{GS} \geq V_T$ and $v_{DS} \geq v_{GS} - V_T$

$$i_{DS} = \frac{K}{2}(v_{GS} - V_T)^2$$

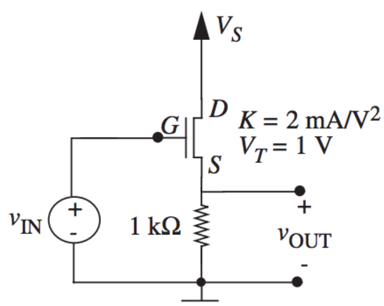
Perform large signal analysis:

1. What is the relationship between v_O and v_{IN} in the saturation region?
2. What is the range of valid input and output values for the source follower?

$$i_{DS} = \frac{v_O}{R_L} = \frac{K}{2}(v_{GS} - V_T)^2$$

$$v_{GS} = v_{IN} - v_O$$
$$i_{DS} = \frac{v_O}{R_L} = \frac{K}{2}(v_{IN} - v_O - V_T)^2$$
$$\frac{v_O}{R_L} = \frac{K}{2}(v_{IN} - v_O - V_T)^2$$

Consider the specific case below, what is v_O and i_{DS} given $v_{IN} = 2V$.



$$i_{DS} = \frac{v_O}{1000} = \frac{2 \times 10^3}{2}(2 - v_O - 1)^2$$
$$v_O^2 - 3v_O + 1 = 0$$
$$v_O = 2.6 \; and \; 0.4 \; V$$

Since $v_{IN} - v_O \geq V_T$ for saturation region,

$$v_O \leq v_{IN} - V_T$$
$$v_O \leq 1$$

$v_O = 0.4 \; V$ and $i_{DS} = 0.4 \; mA$

Find the valid operating ranges for the source follower circuit given $V_S = 10$ V.

For MOSFET to be in the saturation region:

$v_{DS} \geq v_{GS} - V_T$ and $v_{GS} \geq V_T$

In the case of the source follower:

$$v_{GS} \geq V_T$$
$$v_{IN} - v_O \geq V_T$$
$$v_{IN} \geq V_T + v_O$$

Since minimum v_O is 0, then the minimum v_{IN} to allow operation in saturation region:

$$v_{IN} = V_T = 1 \; V$$

Based on the second condition $v_{DS} \geq v_{GS} - V_T$:

$$v_{DS} \geq v_{GS} - V_T$$
$$V_S - v_O \geq v_{IN} - v_O - V_T$$
$$v_{IN} \leq V_S + V_T$$

The maximum v_{IN} to allow operation in saturation region:

$$v_{IN} = V_S + V_T = 10 + 1 = 11 \; V$$

Overall, the valid input voltage range is:
$1V \rightarrow v_{IN} \rightarrow 11V$

The corresponding output voltage at $v_{IN} = 0$ is $v_O = 0$. At $v_{IN} = 11\,V$, v_O is given by:

$$\frac{v_O}{R_L} = \frac{K}{2}(v_{IN} - v_O - V_T)^2$$

$$\frac{v_O}{1000} = \frac{2 \times 10^3}{2}(11 - v_O - 1)^2$$

$$v_O^2 - 21v_O + 100 = 0$$

$$v_O = 13.7\ V \quad and \quad v_O = 7.3\ V$$

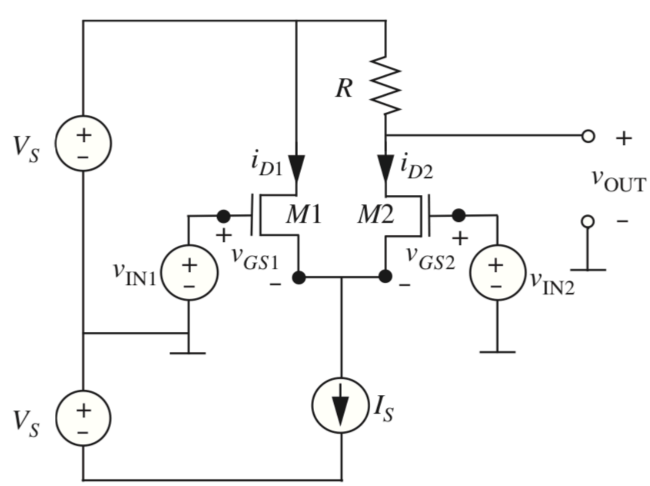
The smaller root ($v_O = 7.3\ V$) is selected since for operation in the saturation region, $v_O \leq v_{IN} - V_T$.

The valid output voltage range is:
$0\,V \rightarrow v_O \rightarrow 7.3\,V$

Given $i_{DS} = \frac{v_O}{R_L} = \frac{v_O}{1000}$, the valid current range is:
$0\,mA \rightarrow i_{DS} \rightarrow 7.3\,mA$

Example: Differential Amplifier

Consider the differential amplifier circuit below. Find v_{OUT} as a function of v_{IN1} and v_{IN2} assuming that both MOSFETs are identical and are operating in saturation.



$$i_{D1} = \frac{K}{2}(v_{GS1} - V_T)^2$$
$$i_{D2} = \frac{K}{2}(v_{GS2} - V_T)^2$$

Apply KCL at the node where the two MOSFETs and the current source join:

$$i_{D1} + i_{D2} = I_S$$

Apply KVL to the loop around the two MOSFETs through the ground:

$$v_{IN1} - v_{GS1} + v_{GS2} - v_{IN2} = 0$$

From the equations above we can find an expression for i_{D2} as a function of v_{IN1} and v_{IN2}.

$$I_S = i_{D2} + \frac{K}{2}\left(v_{IN1} - v_{IN2} + \sqrt{\frac{2i_{D2}}{K}}\right)^2$$
$$2\left(\sqrt{\frac{2i_{D2}}{K}}\right)^2 + 2(v_{IN1} - v_{IN2})\sqrt{\frac{2i_{D2}}{K}} + (v_{IN1} - v_{IN2})^2 - \frac{2I_S}{K} = 0$$
$$i_{D2} = \frac{K}{8}\left(\sqrt{\frac{4I_S}{K} - (v_{IN1} - v_{IN2})^2} - v_{IN1} + v_{IN2}\right)^2$$

Then we can find v_{OUT} using the expression for i_{D2}.

$$i_{D2} = \frac{V_S - v_{OUT}}{R}$$
$$v_{OUT} = V_S - Ri_{D2}$$
$$v_{OUT} = V_S - \frac{RK}{8}\left(\sqrt{\frac{4I_S}{K} - (v_{IN1} - v_{IN2})^2} - v_{IN1} + v_{IN2}\right)^2$$