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6.002 Recitation Notes – Spring 2020

Examples: Nonlinear resistor networks and small signal analysis

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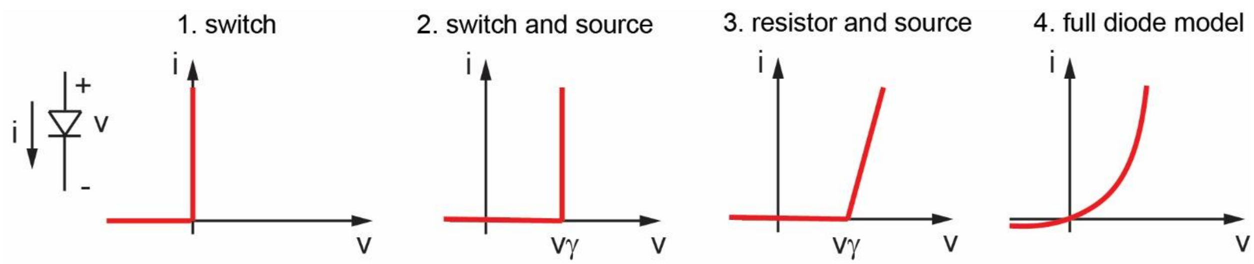
Outline:

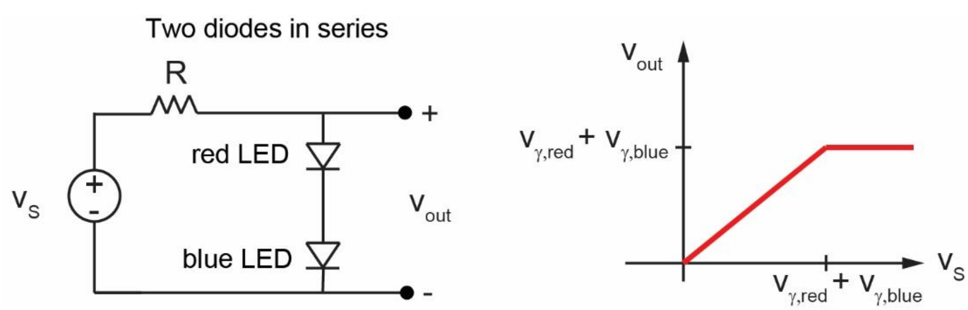
1. Circuit having multiple diodes
2. Zener diode
3. Voltage regulator circuit
4. Regulated current source

1. Example: a circuit that has multiple diodes

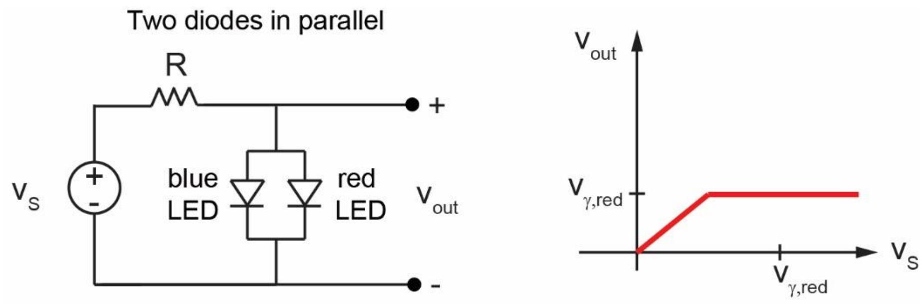
We have learned four diode models in our class, and their i-v characteristics are plotted below.



Note that as model accuracy increases, model complexity also increases. We want to choose an appropriate model for solving a specific problem. In this example, we are going to explore having multiple diodes connected either in series or in parallel. We will use a light emitting diode (LED) as our example. We will choose model 2 (a switch plus a source). Due to differences in the device physics, the turn on voltage v_γ of different colored LEDs are different. For instance, for a blue LED, v_γ is approximately 3 V. For a red LED, v_γ is approximately 1.7 V. Consider the circuit in which a blue LED and a red LED are connected in series. The circuit has a constant voltage source Vs and a series resistor of R=1 kΩ.



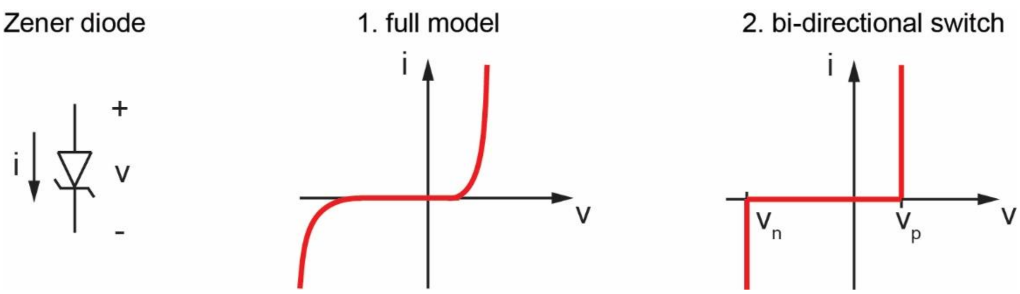
There are two possibilities. If Vs > $v_{\gamma,red} + v_{\gamma,blue} = 4.7\ V$, then both diodes will turn on (both switches will be closed). In contrast, if Vs < $v_{\gamma,red} + v_{\gamma,blue}$, then BOTH diodes will turn off (both switches will be open). The voltage across the diodes will distribute in such a way that none will reach the turn on voltage. In a series configuration, both LEDs will be turned on or off at the same time. Now let's consider a blue LED and a red LED connected in parallel.



It also has two possibilities. If Vs < $v_{\gamma,red}$, then both LEDs will be off. If Vs > $v_{\gamma,red}$, then the red LED will turn on. However, because $v_{\gamma,red} < v_{\gamma,blue}$, the blue LED will never turn on. In practice, we rarely put LEDs in parallel without adding resistors in series.

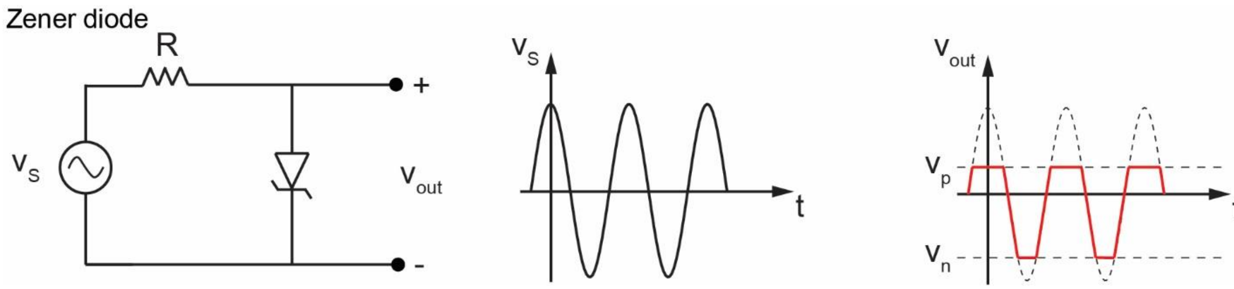
2. Example: a nonlinear component: Zener diode

Now let's look at another nonlinear component, the Zener diode. This component is a "bi-directional" switch. It's i-v characteristic is shown below.

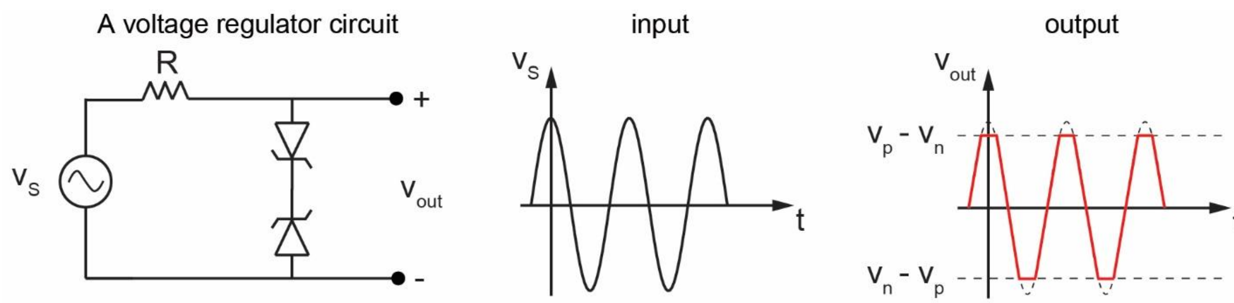


The full model description is highly nonlinear (model 1) and as a consequence obtaining analytical solution is mathematically messy. Instead, we will be focusing on a "bi-directional" switch model. The element is "open" when the applied voltage is in the range [vn, vp]. Here vn is a negative number. The element is a "short" when the applied voltage is in the range [-inf, vn] or [vp, inf]. This component is usually used in a voltage regulator circuit, where we want to limit the maximum and minimum voltage output.

Let us first consider a simple case. In the figure below we connect a Zener diode in series with a sinusoidal voltage source and a resistor. Suppose the input sinusoidal signal has a large range (V_{max} > |vn| and V_{max} > |vp|), then the output signal resembles the red curve shown in the right plot. The output is limited to a maximum of vp and a minimum of vn. However, vp and vn may not have the same magnitude due to the device physics of a Zener diode. In a voltage regulator circuit, we need a design whose range is symmetric with respect to 0.

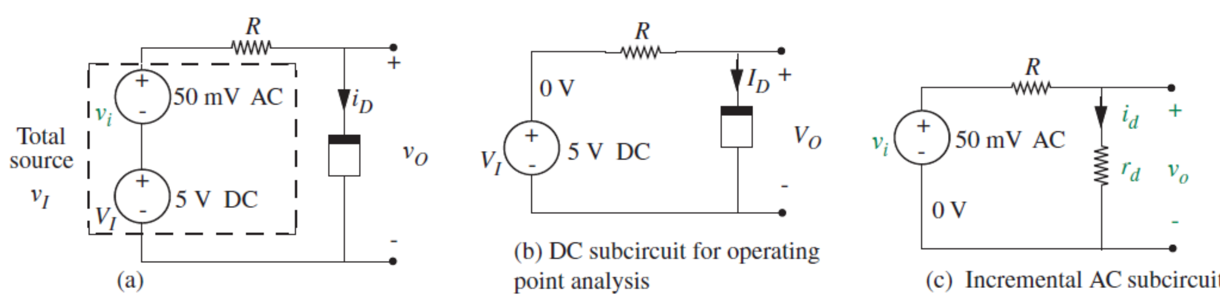


We can achieve this design by putting two Zener diodes in series (and in opposite orientations). When the input voltage signal is in the range of [-(|vp|+|vn|), |vp|+|vn|], the two Zener diodes act as an open circuit. When the instantaneous magnitude of the input voltage signal is larger than |vp |+|vn|, then the two Zener diodes act as a closed circuit (plus voltage sources) so the voltage output is limited. The voltage output resembles the red curve in the right plot. This is an example of a symmetric voltage regulator circuit. It will guarantee that your circuit will not exceed the voltage limit of a load.



3. Example: a voltage regulator circuit (method of small signal analysis)

Let's consider another voltage regulator circuit. The meaning of "voltage regulation" is loosely defined, here it simply means we want the output voltage to resemble certain behaviors. We will follow Example 4.20 in the textbook. Here, "voltage regulation" means we want our circuit to attenuate a noise at the input to a smaller noise at the output.



The figure above (part a) shows the circuit. The i-v characteristic of this nonlinear component is given by:

$$\begin{cases} i_D = K v_D^2 \text{ when } v_D > 0 \\ i_D = 0 \text{ when } v_D < 0 \end{cases}$$

In figure (a), we are interested in driving the circuit with a 5V DC input and a 50mV AC input. Let's think about the 50mV AC input as noise. We aim to find the output v_O. We will use the small signal approach to solve for the output v_O. We use a two-step process:

1. Find the DC operating variables I_D and V_O by setting the small-signal to 0.
2. Linearize around this operating point and find the small-signal v_o and i_d.

The first step is illustrated by (b). Let's solve I_D and V_O using KVL:

$V_I = I_D R + V_O$ and $I_D = K V_O^2$

We have 2 equations and 2 unknowns, and we can rearrange to solve for the variables. Specifically, we can solve for V_O:

$$V_O = \frac{-1 + \sqrt{1 + 4V_I RK}}{2RK}$$

Having solved the large signal V_O, we can now solve for the small signal v_o. We can simplify the circuit to the one shown in (c). Note that we need to solve for the incremental resistance r_d. The small signal analysis (which is derived from Taylor series expansion) gives us the relationship:

$$i_d = \left.\frac{d(K v_O^2)}{d v_O}\right|_{v_O = V_O} v_o$$

This relationship suggests r_d has the form of:

$$r_d = \frac{1}{\left.\frac{d(K v_O^2)}{d v_O}\right|_{v_O = V_O}}$$

Simplifying, we have:

$$r_d = \frac{1}{2KV_O}$$

Next, we can find v_o. The circuit in (c) is a simple voltage divider:

$$v_o = v_i \frac{r_d}{R + r_d} = v_i \frac{1/2KV_O}{R + 1/2KV_O}$$

We have solved for both V_O and v_o. Finally, let's discuss the physical intuition. Let's choose some practical circuit parameter values. Let K = 1 mA/V^2, R = 1 kΩ, V_I= 5 V, and v_i = 50 mV. Substituting these numbers, we have V_O = 1.8V and v_o = 10.9 mV. This circuit is a voltage regulating circuit because we have the relationship:

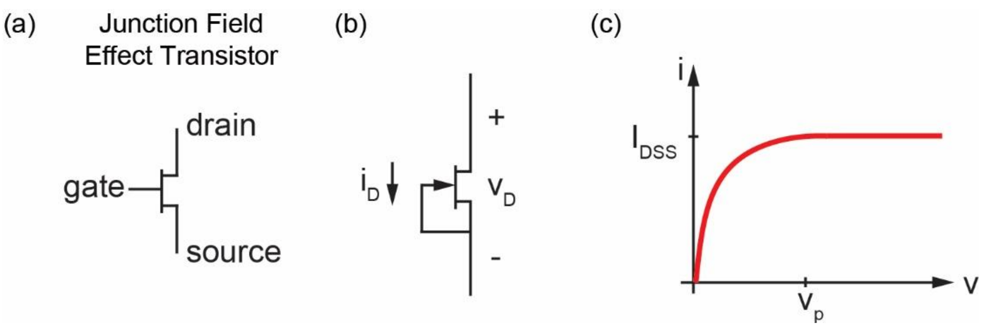
$$\frac{v_i}{V_I} > \frac{v_o}{V_O}$$

This equation means the noise to input ratio at the input end is larger than the ratio at the output end. So even if we have some noise at the input, the relative noise magnitude is attenuated at the output. This is a benefit of a nonlinear circuit, and we solved it using small signal analysis.

4. <u>Example: a regulated current source (method of small signal analysis)</u>

(This example is taken from problem 4.7)

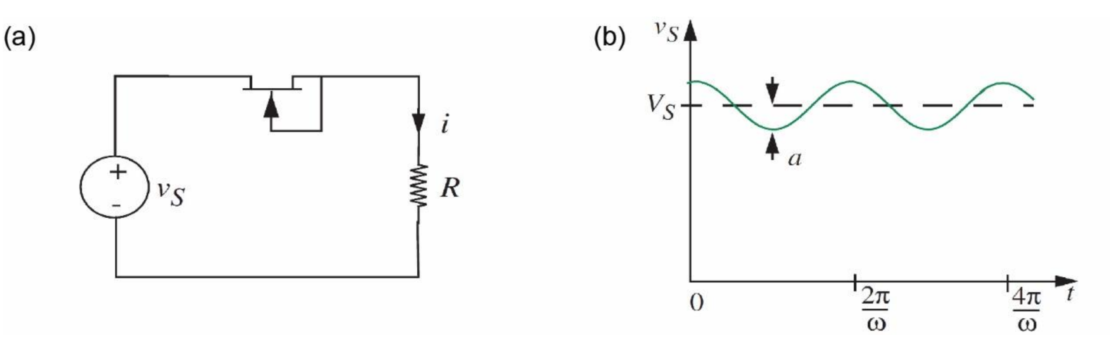
Thus far we focused on two-terminal nonlinear devices. In the following weeks, we will study nonlinear three-terminal devices such as a transistor. Without formally introducing a transistor, let us consider a junction field-effect transistor (JFET). In this case, we will short the transistor's gate terminal to its source terminal and use it as a two-terminal device. The figure below labels the transistor terminals (a), the wiring from source to gate (b), and the device's i-v characteristics.



The device physics show this component has a "pinch-off" voltage vp. The i-v characteristic is given by:

$$i_D = I_{DSS}[2\left(\frac{v_D}{v_P}\right) - \left(\frac{v_D}{v_P}\right)^2] \text{ when } v_D \leq v_P$$
$$i_D = I_{DSS} \text{ when } v_D > v_P$$

For this problem, we only consider the regime of $v_D > 0$.



We are interested in building a regulated current source. That is, even if there is a small noise (oscillation) in the driving voltage, the output current i will have no oscillation. Let us consider the circuit shown above (a). First, we use KVL to solve for the large signal behavior:

$$V_S = V_D + V_R$$

When $V_D \leq v_P$, we have

$$V_S = V_D + I_D R \text{ and } I_D = I_{DSS}\left[2\left(\frac{v_D}{v_P}\right) - \left(\frac{v_D}{v_P}\right)^2\right]$$

We have two equations and 2 unknowns. We can solve for I_D using the quadratic formula.

$$I_D = \frac{1}{2R}\left(2V_S - \left(\frac{v_P^2}{RI_{DSS}} + 2v_P\right) + \sqrt{\left(\frac{v_P^2}{RI_{DSS}} + 2v_P\right)^2 - \frac{4v_P^2 V_S}{RI_{DSS}}}\right)$$

when the condition $V_S - I_D R \leq v_P$ is satisfied.

In the second case, when $V_D > v_P$, which implies $V_S - I_{DSS} R \leq v_P$, we have $I_D = I_{DSS}$. The intuition is that we want to operate the circuit in the "saturated" regime. If our input voltage V_S is large enough such that $I_D = I_{DSS}$, then a small oscillation signal v_s will not cause any change in output current. In the unsaturated regime, however, a small oscillation v_s will cause a non-zero current oscillation i_D. Based on the small signal analysis, we have:

$$i_d = \frac{df(v_O)}{dv_O}\bigg|_{v_O = V_O} v_o$$

Substitute in our solution from I_D, we have

when $V_S - I_D R \leq v_P$,

$$i_d = I_{DSS}\left[\frac{2}{v_P} - \frac{2(V_S - I_D R)}{v_P^2}\right] v_o$$

when $V_S - I_D R > v_P$,

$$i_d = 0$$

This shows that our regulated current source is noise resistant in the saturated regime.