6.002 Recitation Notes – Spring 2020

Boost Converter

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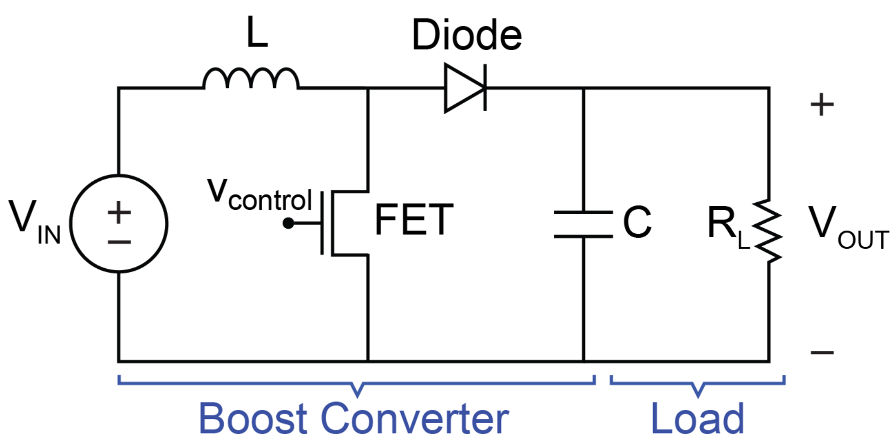
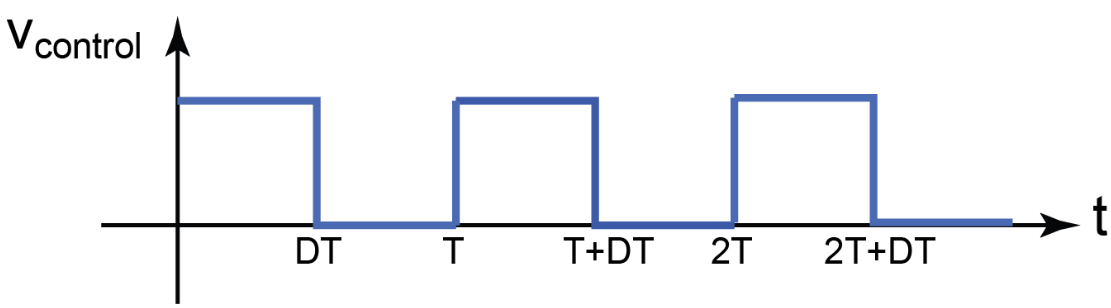
A boost converter, converts a given input DC voltage to a higher output DC voltage. Such a converter is necessary for many applications where the supply voltage is not sufficiently high to run the desired application. Many battery-operated devices benefit from such converters, for example a camera flash, many applications in laptops and smart phones, and hybrid electric vehicles.

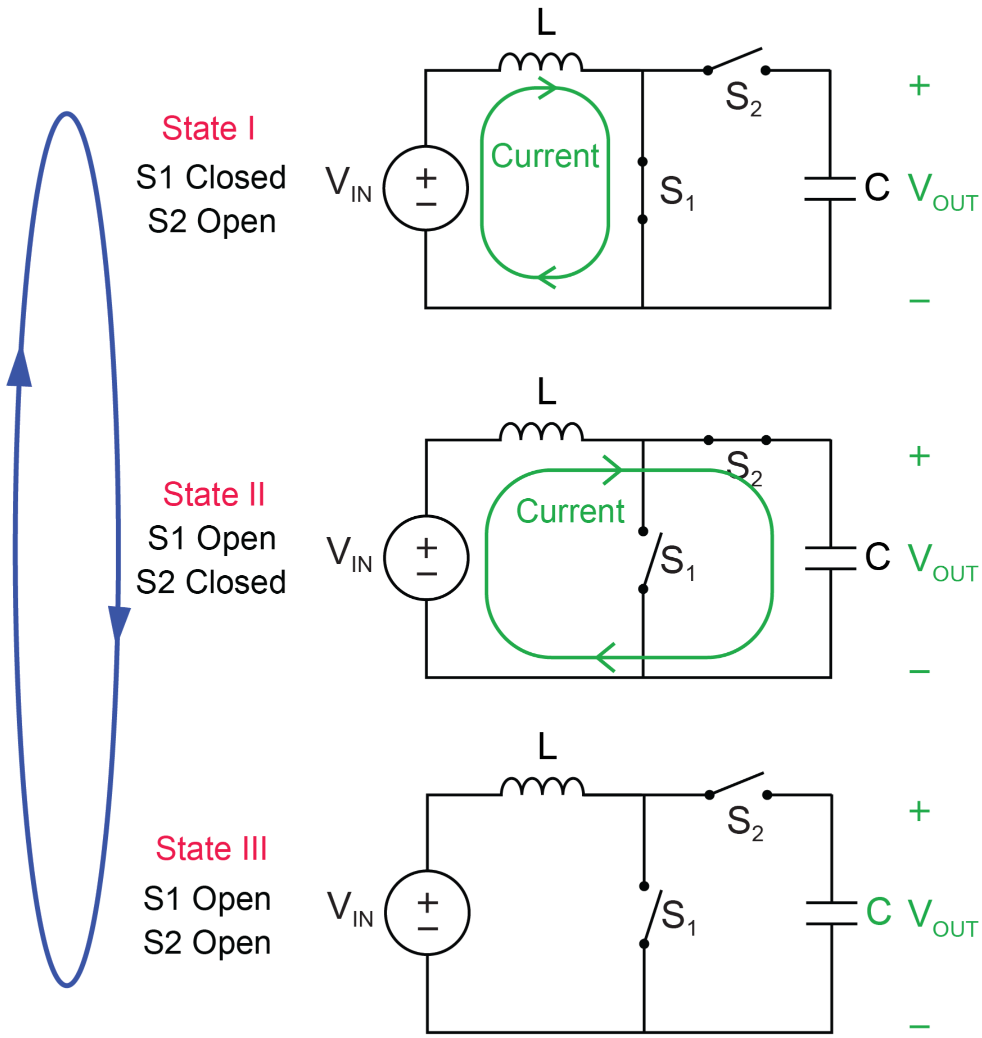
A boost converter consists of an inductor and a capacitor which are energy storing elements, and a transistor and a diode which act like switches whose operation depends on some external conditions.



Depending on the states of the switches, the boost converter circuit goes through a three-stage cyclic operation. Note that the FET's operation is controlled by the $v_{control}$. It is closed when $v_{control}$ is high and open when $v_{control}$ is low. A PWM signal applied to $v_{control}$ is what pushes the system through the three states. This is based on a period T and duty cycle D (varies from 0 to 1).



We will learn more about the transistor and diode operation later on in the term. For now, we can simplify our analysis by replacing them with ideal switches. The three states of operation are summarized below.

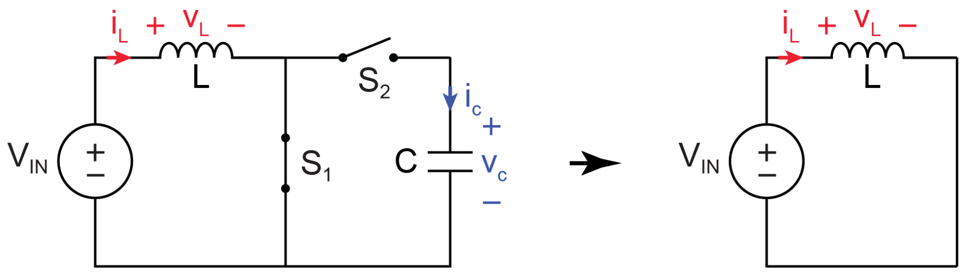


State I: In this stage, S1 is closed and S2 is open. The inductor which is connected to the voltage source stores energy in form of a magnetic field and begins to build up a current through it. While the inductor builds up current (energy), the capacitor on the output is isolated from the rest of the circuit and provides energy to any load that is attached to the system.

State II: In this stage, S1 is open and S2 is closed. During stage I, a current has built up through the inductor. Once S1 is open, current to the inductor stops abruptly. However, inductor resists abrupt changes in current (current through the inductor is continuous). This causes the current to flow through S2, the capacitor charges up and the voltage across it builds up. In the process of charging up the capacitor, the voltage across the inductor will decrease and results in a drop in the inductor current.

State III: In this stage, S1 and S2 are both open. As long as the current through the inductor is positive, S2 stays closed letting current to flow through the capacitor. When the current gets low enough, S2 will open (stage III). At this stage, the capacitor is isolated from the left side and the energy stored in the capacitor can be dissipated through any load that is attached to it.

State I



$-\mathrm{T} \leq \mathrm{t} \leq 0$

$i_L(-\mathrm{T}) = 0$

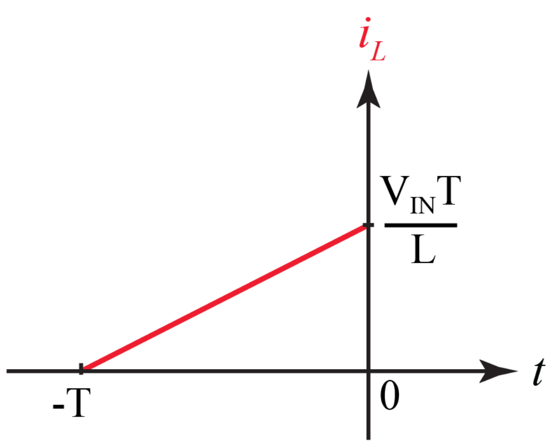
$v_C(-\mathrm{T}) = \mathrm{Given}$

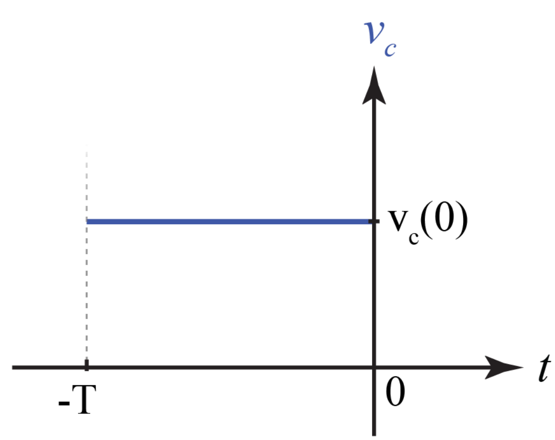
During this stage a current is building up in the inductor and the capacitor is isolated from the rest of the circuit.

$$v_L(t) = L\frac{di_L(t)}{dt} = V_{IN} \quad \Longrightarrow \quad i_L(t) = \frac{1}{L}\int_{-T}^{t} V_{IN}\, dt = \frac{V_{IN}}{L}(t+T)$$

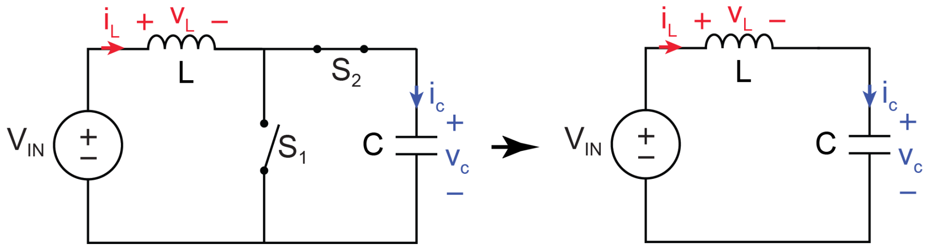
$$i_C(t) = C\frac{dv_C(t)}{dt} = 0 \quad \Longrightarrow \quad v_C(t) = v_C(-T)$$

The following plots show $i_L(t)$ and $v_C(t)$ for $-T \leq t \leq 0$.





State II



$0 \le \mathrm{t} \le \mathrm{S2\ opened}$

$i_L(0) = \mathrm{Given}$

$v_C(0) = \mathrm{Given}$

The current built up in the inductor is directed to the capacitor, charging the capacitor and building up the voltage across it. This is a driven second-order LC circuit, the same as the example covered in the lecture. We will briefly review the operation here.

Use KVL (or KCL) to write a second-order differential equation describing the system.

$$V_{IN} = v_L + v_C$$

$V_{IN} = L\frac{di_L}{dt} + v_C$ but note that $i_L = i_c$ and $i_c = C\frac{dv_C}{dt}$

$$V_{IN} = LC\frac{d^2 v_C}{dt^2} + v_C$$

$$\boldsymbol{\frac{d^2 v_C}{dt^2} + \frac{1}{LC}v_C = \frac{V_{IN}}{LC}}$$ **Equation (1)**

To solver for $v_C(t)$, follow these three steps:

1. Find the homogenous solution.
2. Find the particular solution.
3. The total solution is the sum of the homogeneous and particular solutions. Use the initial conditions to solve for the remaining constants.

Homogeneous Solution

Find the homogeneous solution. The solution is of the general form Ae^{st} where $s = j\omega$. Substitute this into Equation 1.

$$\frac{d^2(Ae^{st})}{dt^2} + \frac{1}{LC}Ae^{st} = 0$$

$$As^2e^{st} + \frac{1}{LC}Ae^{st} = 0$$

$$Ae^{st}\left(s^2 + \frac{1}{LC}\right) = 0$$

$$s^2 + \frac{1}{LC} = 0$$

$s = \pm j\sqrt{\frac{1}{LC}} = \pm j\omega_0$ where $\omega_0 = \frac{1}{\sqrt{LC}}$.

The solution is then the linear combination of $A_1 e^{j\omega_0 t}$ and $A_2 e^{-j\omega_0 t}$.

$v_{CH} = A_1 e^{j\omega_0 t} + \ A_2 e^{-j\omega_0 t}$

This can also be written as:

$v_{CH}(t) = K_1 \cos\,(\omega_0 t) + K_2 \sin\,(\omega_0 t)$

Particular Solution

Before finding the constants K_1 and K_2, we need to find the particular solution and add it to v_{CH} to find the total solution.

$$\frac{d^2 v_{CP}}{dt^2} + \frac{1}{LC} v_{CP} = \frac{V_{IN}}{LC}$$

The input is a DC voltage source and hence V_{IN} is constant. A particular solution that is a constant will satisfy the above differential equation, such that:

$v_{CP}(t) = V_{IN}$

Total Solution

The total solution is the sum of the homogeneous and particular solutions.

$v_C(t) = v_{CH}(t) + v_{CP}(t)$

$\boldsymbol{v_C(t) = K_1 \cos(\omega_0 t) + K_2 \sin(\omega_0 t) + V_{IN}}$

Based on this expression we can also find $i_L(t)$ noting that $i_L(t) = i_C(t) = C\frac{dv_C(t)}{dt}$.

$i_L(t) = C[-K_1\,\omega_0 \sin(\omega_0 t) + K_2\,\omega_0 \cos(\omega_0 t)]$

$$\boldsymbol{i_L(t) = -K_1\sqrt{\frac{C}{L}}\sin(\omega_0 t) + K_2\sqrt{\frac{C}{L}}\cos(\omega_0 t)}$$

Now, solve for K_1 and K_2 using the initial conditions: $v_C(0)$ and $i_L(0) = i_C(0) = C\frac{dv_C(0)}{dt}$

$v_C(0) = K_1 + V_{IN}$

$K_1 = v_C(0) - V_{IN}$

and,

$$i_L(0) = K_2\sqrt{\frac{C}{L}}$$

$$K_2 = i_L(0)\sqrt{\frac{L}{C}}$$

Substituting in for K_1 and K_2 gives the following expressions.

$$v_C(t) = (v_C(0) - V_{IN})\cos(\omega_0 t) + i_L(0)\sqrt{\frac{L}{C}}\sin(\omega_0 t) + V_{IN}$$

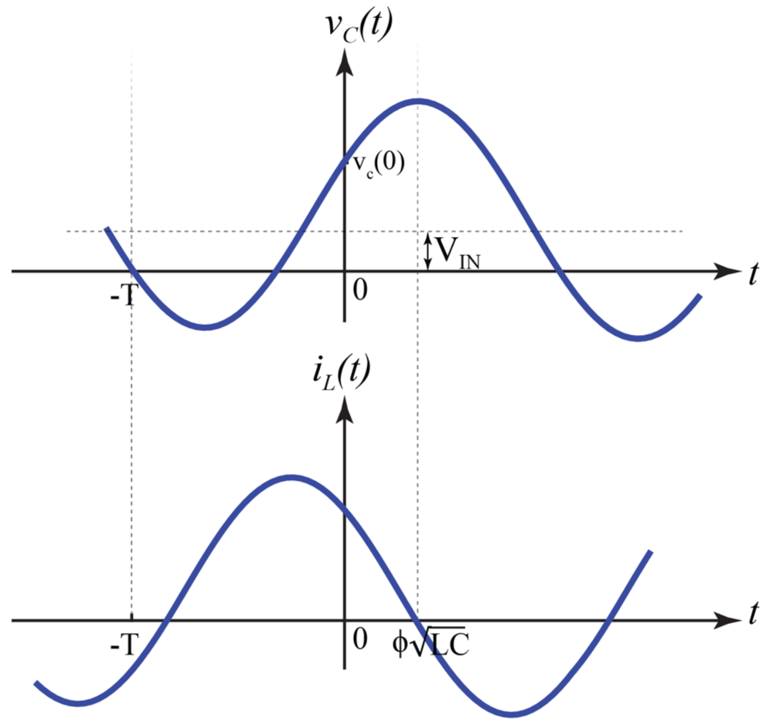
$$i_L(t) = -(v_C(0) - V_{IN})\sqrt{\frac{C}{L}}\sin(\omega_0 t) + i_L(0)\cos(\omega_0 t)$$

By applying some trigonometric identities, these expressions can be re-written as shown below.

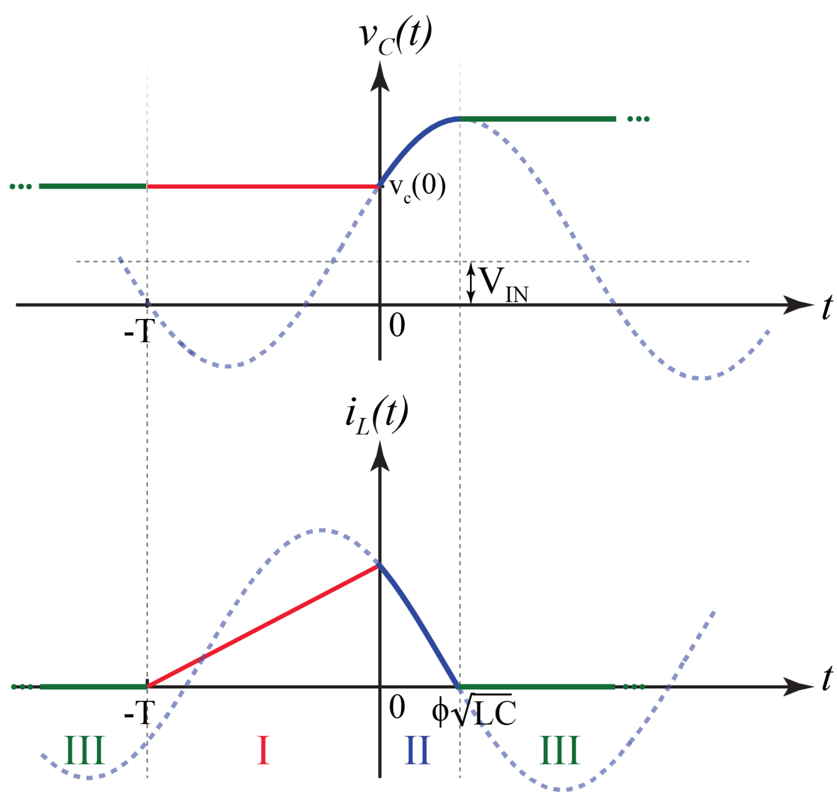
$$v_C(t) = V_{IN} + \sqrt{(v_C(0) - V_{IN})^2 + \frac{L}{C}i_L^2(0)}\cos\left(\frac{t}{\sqrt{LC}} - \overbrace{\tan^{-1}\left(\sqrt{\frac{L}{C}}\,\frac{i_L(0)}{v_C(0) - V_{IN}}\right)}^{\phi}\right)$$

$$i_L(t) = -\sqrt{\frac{C}{L}(v_C(0) - V_{IN})^2 + i_L^2(0)}\sin\left(\frac{t}{\sqrt{LC}} - \tan^{-1}\left(\sqrt{\frac{L}{C}}\,\frac{i_L(0)}{v_C(0) - V_{IN}}\right)\right)$$

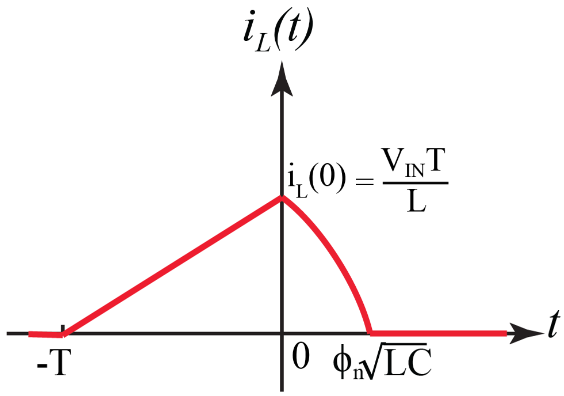
Note that i_L and v_C are sinusoidal functions that are out of phase with each other.

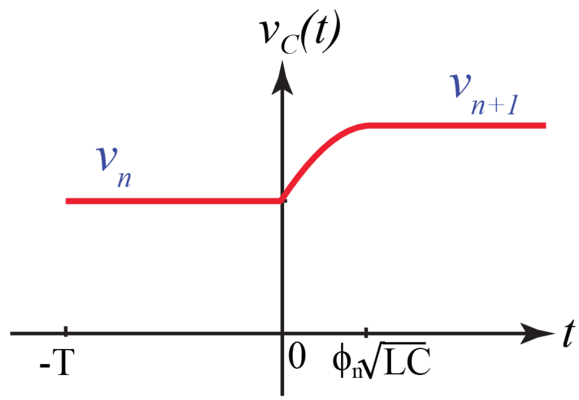


Now lets consider the three states of the boost converter. The expected performance during the different states is shown below.



Cycle Analysis





$$v_{n+1} = V_{IN} + \sqrt{(v_n - V_{IN})^2 + \frac{L}{C} i_L^2(0)}$$

$$(v_{n+1} - V_{IN})^2 = (v_n - V_{IN})^2 + \frac{L}{C} i_L^2(0)$$

$$(v_{n+1} - V_{IN})^2 = (v_n - V_{IN})^2 + \frac{V_{IN}^2 T^2}{LC}$$

Considering that $(v_0 - V_{IN})^2 \equiv 0$, then:

$$v_n = V_{IN} + V_{IN}\sqrt{\frac{nT^2}{LC}}$$

Cycle Analysis Using Energy Conservation

Conservation	$\Delta W_C = \Delta W_B + W_L$
Capacitor	$\Delta W_C = \frac{1}{2} C v_{n+1}^2 - \frac{1}{2} C v_n^2$
Inductor	$W_L = \frac{1}{2} L i_L^2(0)$
DC Voltage Supply (Battery)	$\Delta W_B = V_{IN} \Delta \mathrm{Q}$
Charge	$\Delta \mathrm{Q} = \mathrm{C} v_{n+1} - C v_n$

$$\Delta W_C = \Delta W_B + W_L$$

$$\frac{1}{2}Cv_{n+1}^2 - \frac{1}{2}Cv_n^2 = CV_{IN}v_{n+1} - CV_{IN}v_n + \frac{1}{2}Li_L^2(0)$$

$$v_{n+1}^2 - 2V_{IN}v_{n+1} + V_{IN}^2 = v_n^2 - 2V_{IN}v_n + V_{IN}^2 + \frac{L}{C}i_L^2(0)$$

$$(v_{n+1} - V_{IN})^2 = (v_n - V_{IN})^2 + \frac{L}{C}i_L^2(0)$$

This is the same as what we got before.

As we go through the switching cycles, the voltage across the capacitor builds up to a larger value. A schematic is shown here.