6.002 Recitation 2

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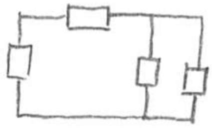
Outline:
- node analysis
- superposition
- Thevenin & Norton equivalence

Last week: KVL & KCL : from energy and charge conservation

↳ a circuit with N elements needs 2N equations (v, i)

e.g.  4 circuit elements → 8 equations

overall goal of this week:

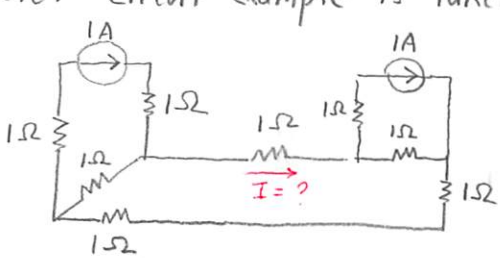
reduce the number of equations when solving a circuit problem.

Tool 1: Node analysis

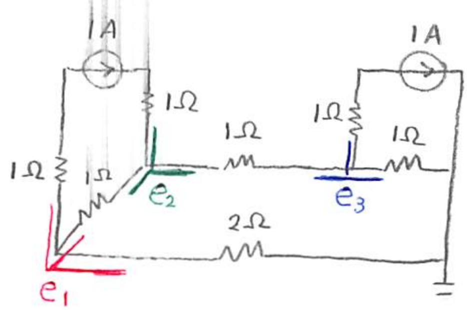
six steps: (refer to lecture 2 notes)

1. draw circuit (and simplify when needed)
2. select a "ground" node
3. label all (essential) remaining un-sourced node voltages with respect to ground
4. write KCL for these nodes, and substitute device laws and KVL
5. solve KCL equations
6. back solve for branch variables (v, i)

Ex: note: circuit example is taken from Ex 3.2.2 (Agarwal & Lang. p163)

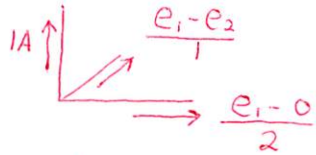


1) simplify the circuit
2) select ground node
3) label essential node voltages



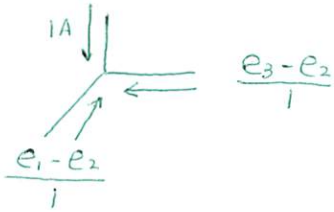
② 4) write KCL :

node 1 :



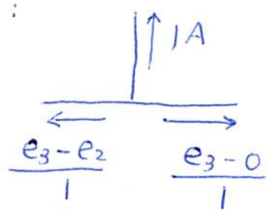
$$1 + \frac{e_1 - e_2}{1} + \frac{e_1 - 0}{2} = 0$$

node 2 :



$$1 + \frac{e_1 - e_2}{1} + \frac{e_3 - e_2}{1} = 0$$

node 3 :



$$1 + \frac{e_3 - e_2}{1} + \frac{e_3 - 0}{1} = 0$$

5) simplify & solve :

$$\left.\begin{aligned} 1 + \tfrac{3}{2} e_1 - e_2 \quad\quad &= 0 \\ 1 + e_1 - 2e_2 + e_3 &= 0 \\ 1 \quad\quad - e_2 + 2e_3 &= 0 \end{aligned}\right\} \Rightarrow \underbrace{\begin{bmatrix} 3/2 & -1 & 0 \\ 1 & -2 & 1 \\ 0 & -1 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}}_{b}$$

$$x = A^{-1}b = [-0.8, -0.2, -0.6]^T$$

6) back solve :

$$I = \frac{e_2 - e_3}{1\Omega} = \frac{-0.2 - (-0.6)}{1\Omega} = \frac{2}{5} A$$

Tool 2 : Superposition

linearity : $f(ax+by) = af(x) + bf(y)$

insight : for a linear circuit, (consisted of resistor, capacitor, and inductor), we can solve two (or more) simpler problems and add their results to solve a complex problem

caveat : linear quantities : v, i

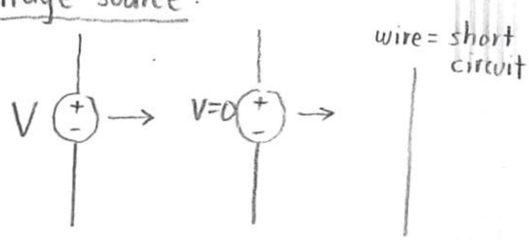
nonlinear quantity : power, energy, etc.

e.g. power = $v \cdot i$ (note : $(V_1+V_2)(i_1+i_2) \neq V_1 i_1 + V_2 i_2$)

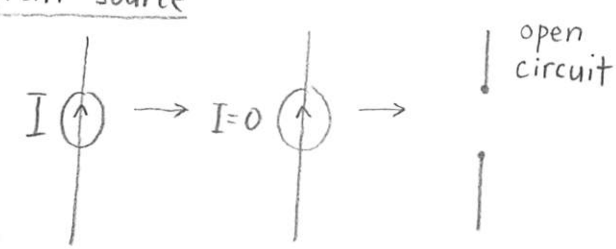
Procedure : turn off all but one independent sources and solve for each sub-circuit

↳ how to turn off an independent source :

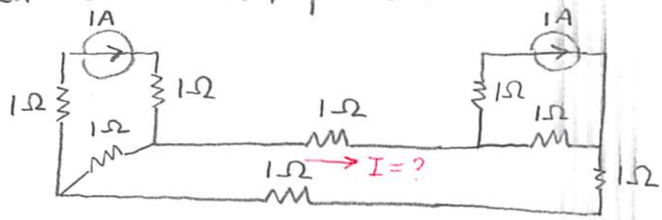
Voltage source :



Current source

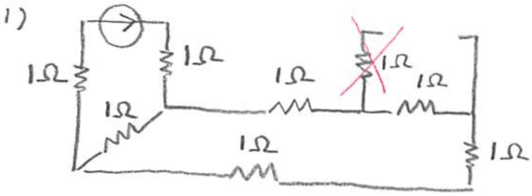


③ Ex : same circuit problem

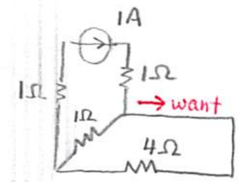


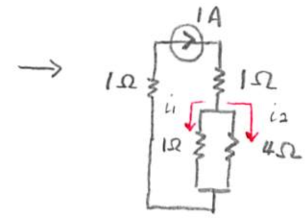
sub-problem 1 : turn off right current source

1)



2) simplify :





3) apply KVL and KCL locally :

KVL : $i_1 \cdot 1 + -i_2 4 = 0$ KCL : $i_1 + i_2 = 1$

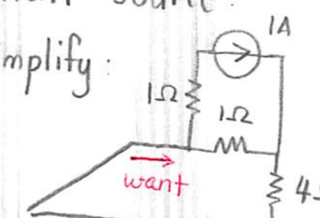
4) solve : $i_1 = \frac{4}{5} A$

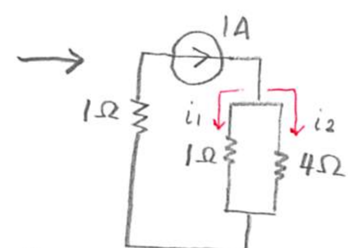
$i_2 = \frac{1}{5} A$

sub-problem 2 : turn off left current source :

1)

2) simplify :





3) apply KVL and KCL locally :

KVL : $i_1 \cdot 1 - i_2 \cdot 4 = 0$ KCL : $i_1 + i_2 = 1$

4) Solve : $i_1 = \frac{4}{5} A$

$i_2 = \frac{1}{5} A$

Sum solutions from sub-problem 1 and sub-problem 2 :

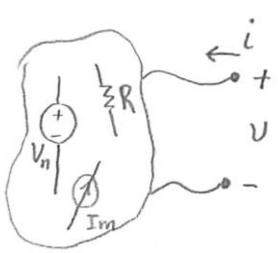
$i_{total} = i_{2,right} + i_{2,left} = 1/5 + 1/5 = 2/5 \text{ A}$

Tool 3 : Thevenin & Norton equivalence

goal : simplify a complex linear circuit to 2 circuit elements

Thevenin circuit : voltage source + resistor

Norton circuit : current source + resistor

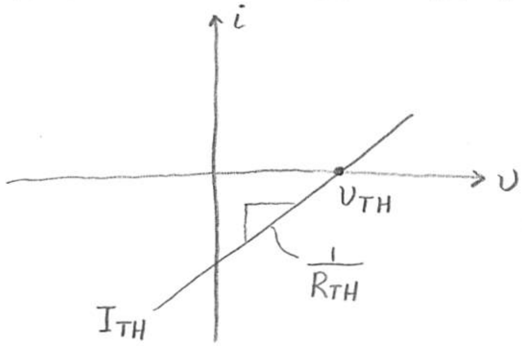


V_{TH} = open circuit voltage at the port

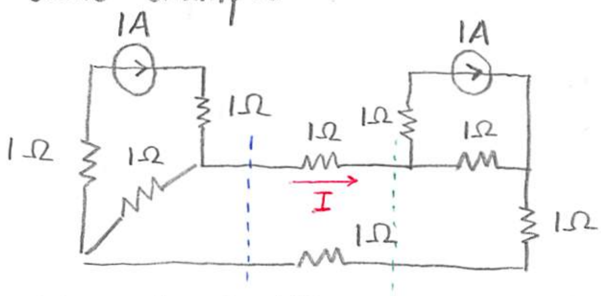
R_{TH} = net resistance looking from the port

I_{TH} = short circuit current at the port

④ motivation: For a linear circuit, 2 values define the i-v characteristics

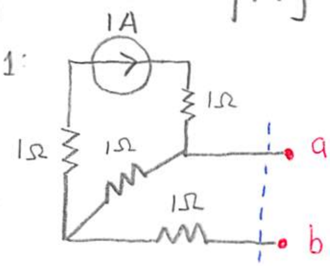


Ex: same example

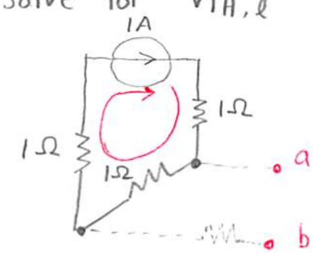


1) split and simplify:

part 1:

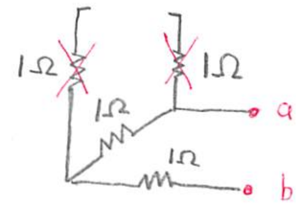


• solve for $V_{TH,l}$



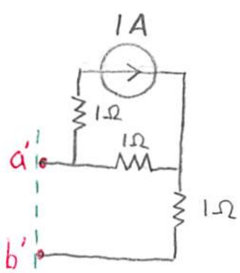
$V_{TH,l} = V_{ab} = 1A \cdot 1\Omega = 1V$

• solve for $R_{TH,l}$

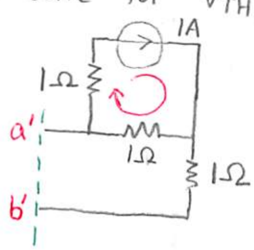


$R_{TH,l} = 1\Omega + 1\Omega = 2\Omega$

part 2:



• solve for $V_{TH,r}$

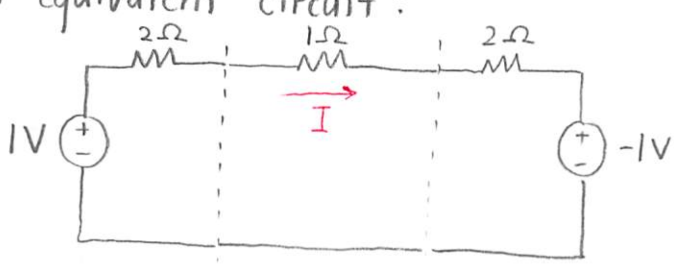


$V_{TH,r} = V_{a'b'} = -1A \cdot 1\Omega = -1V$

• solve for $R_{TH,r}$

$R_{TH,r} = 1\Omega + 1\Omega = 2\Omega$

2) draw an equivalent circuit:



$$I = \frac{V_{net}}{R_{net}} = \frac{1-(-1)}{2+1+2} = \frac{2}{5}A$$

⇒ all three methods give the same result!