6.002 CIRCUITS AND ELECTRONICS

Lecture 24 – **Three transistors amplifier… Let's make an op-amp!**

May 12, 2020

Contents:

1. Differential amplifier: motivation
2. MOSFET differential amplifier
3. Amplifier biasing
4. Beyond 6.002

Reading Assignment:
Agarwal and Lang, Ch. 7 (examples 7.19-7.22), Ch. 8 (example 8.3)

Handouts:
Lecture 24 notes

Announcements:

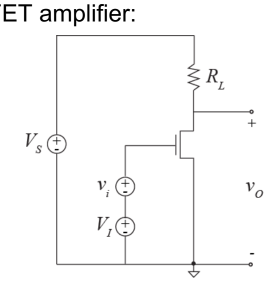
•Final exam (5/18 in Gradescope) will focus on sinusoidal steady state and non-linear devices, however it will build up on all the other topics of 6.002, so please review the content of all psets and quizzes.

•This lecture is being recorded and it will be posted in the certificates-protected part of the 6.002 website.

•Voluntary (not graded) HW 10 can be found in website.

1. Differential amplifier: motivation

Remember MOSFET amplifier:



Signal at output under small-signal approximation:

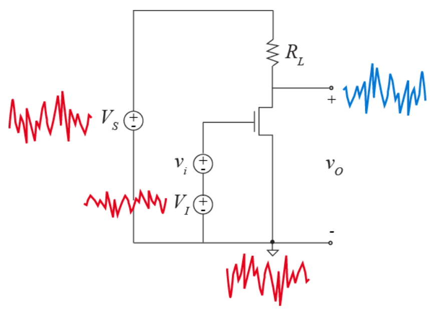
$$v_O = V_O + v_o$$
$$\simeq V_S - \frac{KR_L}{2}(V_I - V_T)^2 - KR_L(V_I - V_T)v_i$$

Output signal depends on:

– power supply and signal bias
– input signal.

In circuits, there is often:

- power supply noise and ground noise (cross talk between metal lines, 60 Hz coupling, Si substrate noise)
- Signal bias is often also noisy



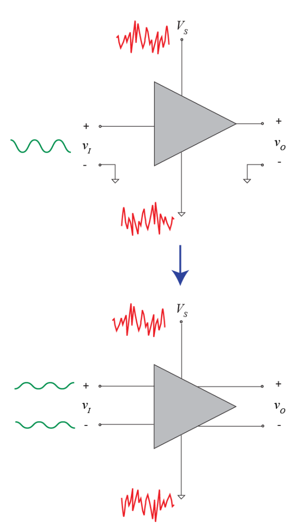
Consequence: noisy output!

→signal can be lost

$$\begin{aligned} v_O &= V_O + v_o \\ &\simeq V_S - \frac{KR_L}{2}(V_I - V_T)^2 - KR_L(V_I - V_T)v_i \end{aligned}$$

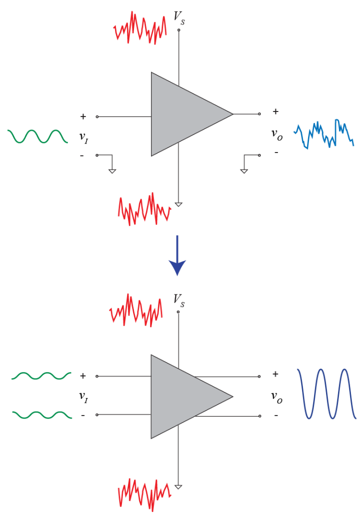
Solution:

- Instead of representing signal by a voltage referred to a fixed reference voltage
- represent signal by the *difference* between two voltages



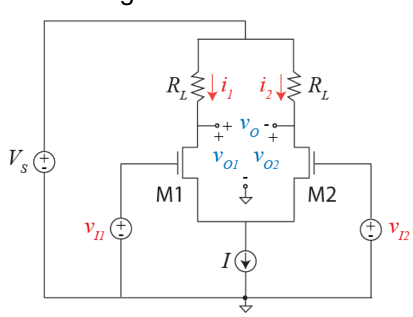
Develop a *differential amplifier* that:

- amplifies voltage *difference* between two inputs
- rejects voltage component *common* to both inputs



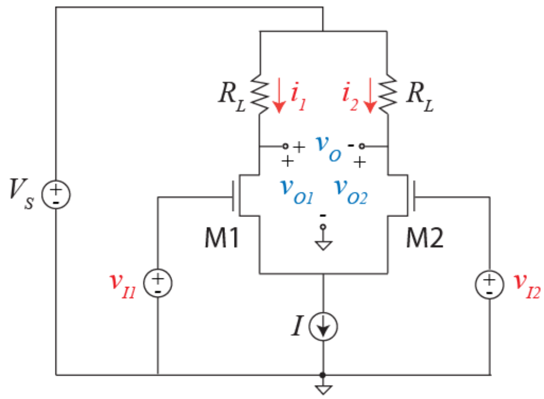
2. MOSFET differential amplifier

Consider the following circuit:



v_O responds to difference of v_I's:

- If $v_{I1}=v_{I2}$ → symmetry
  → $v_{O1}=v_{O2}$ → $v_O=0$
- If $v_{I1}>v_{I2}$ → M1 more ON than M2 → $i_1 > i_2$
  → $v_{O1}<v_{O2}$ → $v_O<0$
- If $v_{I1}<v_{I2}$ → M1 less ON than M2 → $i_1 < i_2$
  → $v_{O1}>v_{O2}$ → $v_O>0$

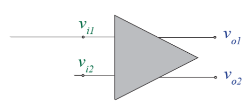


v_O insensitive to *common mode* of v_I:

- If noise in common mode of v_{I1} and v_{I2}
  → symmetry preserved → v_O unchanged
- If ground or V_S have noise, symmetry preserved
  → v_O unchanged

Need precise device matching

In small-signal regime, define differential-mode vs. common-mode signals:



Differential amplifier inputs:

$$v_{i1} = v_{ic} + \frac{v_{id}}{2} \qquad v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

Then:

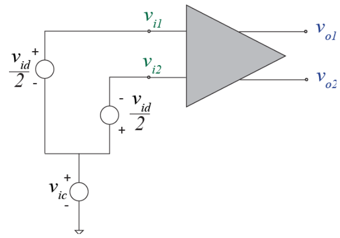
$$v_{id} = v_{i1} - v_{i2} \qquad v_{ic} = \frac{v_{i1} + v_{i2}}{2}$$

Similarly at output:

$$v_{o1} = v_{oc} + \frac{v_{od}}{2} \qquad v_{o2} = v_{oc} - \frac{v_{od}}{2}$$

And:

$$v_{od} = v_{o1} - v_{o2} \qquad v_{oc} = \frac{v_{o1} + v_{o2}}{2}$$



Figures of merit of interest:

- Differential-mode gain (want high):

$$G_d = \frac{v_{od}}{v_{id}}$$

- Common-mode gain (want small)

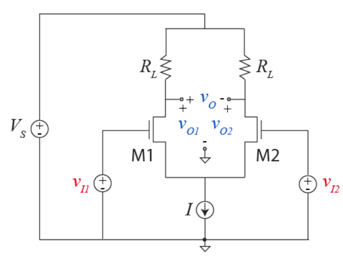
$$G_c = \frac{v_{oc}}{v_{ic}}$$

- Common-mode rejection ratio (want high):

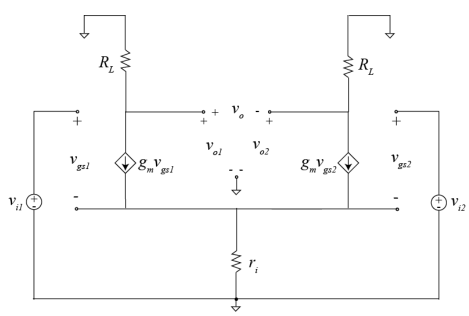
$$CMRR = \frac{G_d}{G_c}$$

Let's calculate all these important parameters!!

Small-signal equivalent circuit for MOSFET differential amp



$$v_{i1} = v_{ic} + \frac{v_{id}}{2}$$

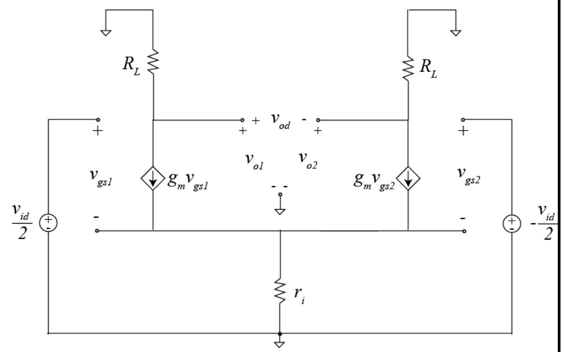


$$v_{i2} = v_{ic} - \frac{v_{id}}{2}$$

- Differential-mode gain:

$$v_{o1} = -g_m R_L v_{gs1}$$

$$v_{o2} = -g_m R_L v_{gs2}$$



Source of M1, M2 is "incremental ground" and, therefore,

$$v_{gs1} = \frac{v_{id}}{2} \qquad v_{gs2} = \frac{-v_{id}}{2}$$

Then:

$$v_{od} \;=\; v_{o1} - v_{o2} \;=\; -g_m R_L \frac{v_{id}}{2} + g_m R_L \left(\frac{-v_{id}}{2}\right) = -g_m R_L v_{id}$$

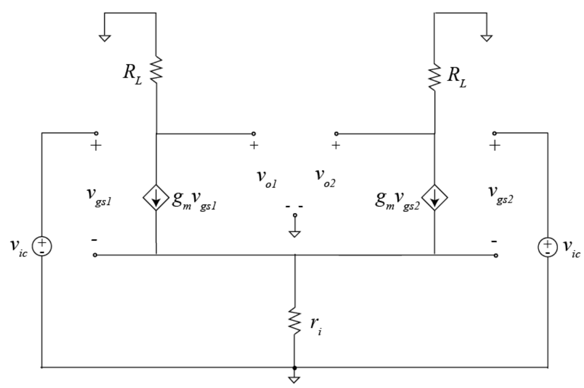
Differential gain:

$$G_d = \frac{v_{od}}{v_{id}} = -g_m R_L$$

- Common-mode gain:

$$v_{o1} = -g_m R_L v_{gs1}$$

$$v_{o2} = -g_m R_L v_{gs2}$$

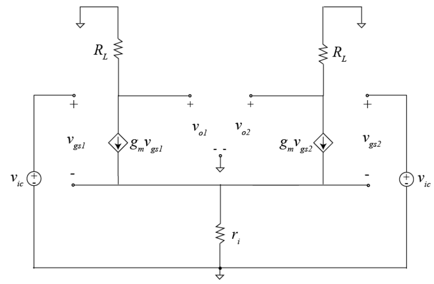


By symmetry:

$$v_{gs1} = v_{gs2} \;\longrightarrow\; v_{o1} = v_{o2}$$

Common-mode output:

$$v_{oc} = \frac{1}{2}(v_{o1} + v_{o2}) = v_{o1}$$



KVL at input:

$$v_{ic} = v_{gs1} + (g_m v_{gs1} + g_m v_{gs2})r_i = (1 + 2g_m r_i)v_{gs1}$$

Then:

$$v_{gs1} = \frac{v_{ic}}{1 + 2g_m r_i}$$

And:

$$v_{oc} = v_{o1} = -g_m R_L v_{gs1} = -\frac{g_m R_L}{1 + 2g_m r_i} v_{ic}$$

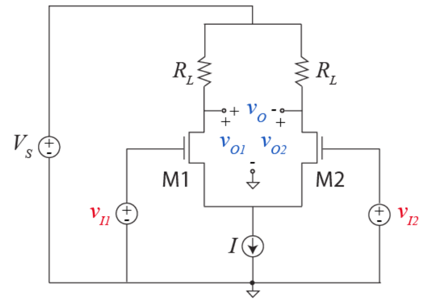
Common-mode gain:

$$G_c = \frac{v_{oc}}{v_{ic}} = -\frac{g_m R_L}{1 + 2g_m r_i}$$

- Common-mode rejection:

$$CMRR = \frac{G_d}{G_c} = 1 + 2g_m r_i$$

Key for common-mode rejection: use a good current source with high r_i



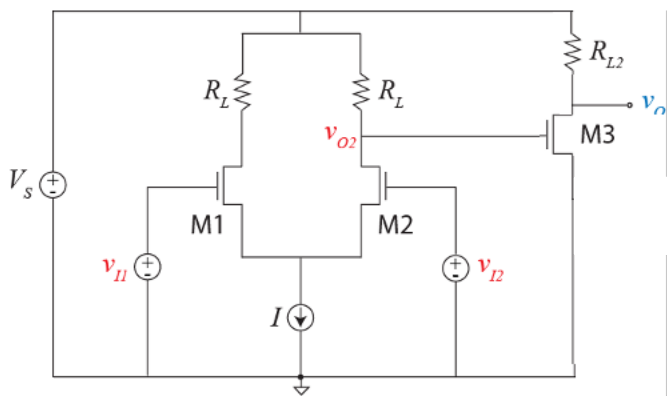
Demo

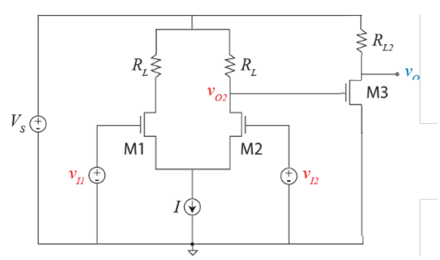
The higher r_i, the smaller the change in I as a result of a common-mode signal
→ the smaller the change in v_{O1} and v_{O2}

Towards an operational amplifier

Add "single-ended" MOSFET amplifier at output:

- more gain
- single-ended output
- can design to get $v_O=0$ when $v_{I1}=v_{I2}$





"Single-ended" gain expressed in terms of differential and common-mode gains:

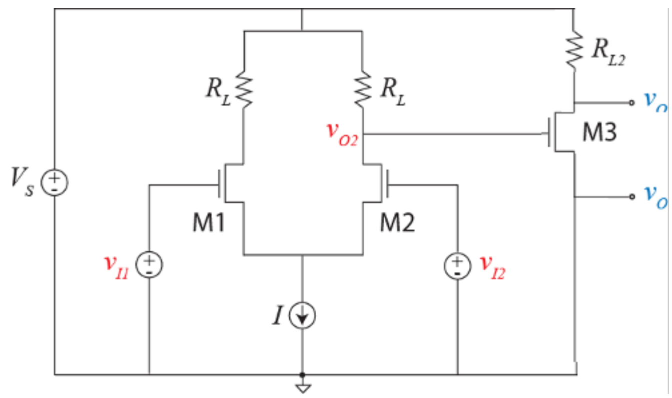
$$\begin{aligned} v_{o2} &= v_{oc} - \frac{v_{od}}{2} = G_c v_{ic} - G_d \frac{v_{id}}{2} \\ &= G_c \frac{v_{i1} + v_{i2}}{2} - G_d \frac{v_{i1} - v_{i2}}{2} \\ &= \frac{1}{2}(G_c - G_d) v_{i1} + \frac{1}{2}(G_c + G_d) v_{i2} \end{aligned}$$

If $G_d >> G_c$:

$$v_{o2} \simeq -\frac{1}{2} G_d (v_{i1} - v_{i2}) = -\frac{1}{2} G_d v_{id}$$

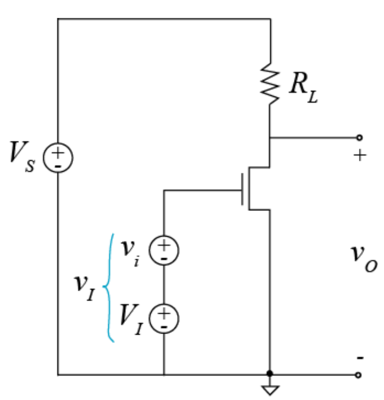
3. Amplifier biasing

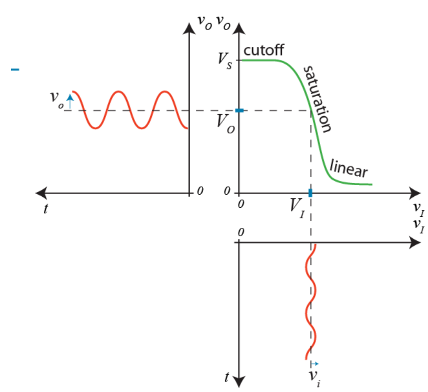
- How do we apply a small signal on top of a large signal bias voltage?



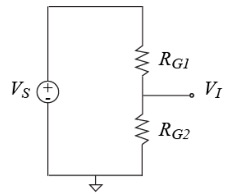
Many times we want the different stages in our circuit to be independent from each other.

- How do we apply a small signal on top of a large signal bias voltage?



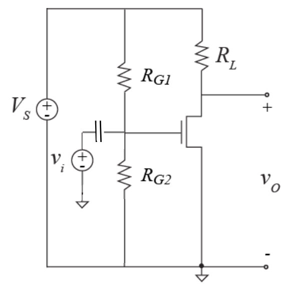


- Problem: need two power supplies, V_S and V_I
- But, can produce V_I out of V_S using voltage divider!

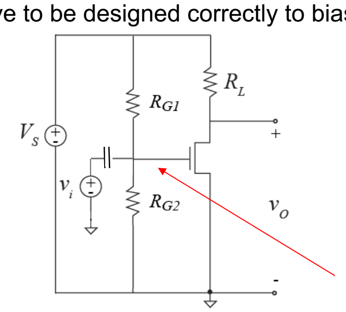


$$V_I = V_S \frac{R_{G2}}{R_{G1} + R_{G2}}$$

- And connecting everything to the circuit:



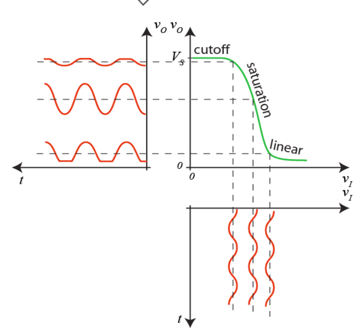
- Resistor values have to be designed correctly to bias MOSFET in saturation



$V_G = V_S \frac{R_{G2}}{R_{G1} + R_{G2}}$

input bias too low

input bias too high



Summary

- Differential amplifier:
  - amplifies voltage difference between two inputs,
  - rejects voltage common to both inputs

  → achieves greater noise immunity.
- Differential amplifier exploits symmetry: need good matching between components
- In MOSFET differential amplifier, to achieve high common-mode rejection, need high quality current source with very high internal resistance

Key lessons from 6.002

- Learned basics of analog, logic and power circuits with various devices:
  - Active and passive devices
  - Dissipative and energy storage devices
  - Linear and non-linear devices
- Learned various techniques to analyze circuits depending on the situation
- Learned to think and work in the frequency domain
  - Concept of impedance
- Useful concepts sprinkled along the way:
  - Modular design
  - Noise immunity
  - Concern for power consumption
  - Concern for device variations and matching
  - Temperature stability
- Gained appreciation for the power of math:
  - Complex algebra
  - Differential equations
- Cannot go very far without detailed physical understanding

- These are all lessons that go much beyond EE

  → big lessons for engineering

What comes after 6.002?

6.003: Signals and Systems

- Fundamentals of signal and system analysis

6.012: Microelectronic Devices and Circuits

- Semiconductor device physics, microelectronic circuit analysis and design

6.101: Introductory Analog Electronics Laboratory
6.111: Introductory Digital Systems Laboratory
6.115: Microcomputer Project Laboratory
6.131: Power Electronics Laboratory

6.301: Solid-State Circuits
6.302: Feedback Systems
6.334: Power Electronics

6.374: Analysis and Design of Digital Integrated Circuits

Good luck with everything!!!

(and let me know if you are interested in a UROP/SuperUROP project and/or helping with the VLSI 2020 conference!)