6.002 CIRCUITS AND ELECTRONICS

Lecture 22 – **Amplifier biasing and small-signal analysis**

May 5, 2020

Contents:

1. Review of MOSFET
2. MOSFET amplifier (cont.)
3. Small-signal analysis
4. Amplifier biasing

Reading Assignment:

Agarwal and Lang, Ch. 7 (§ § 7.7-7.9), Ch. 8 (§ § 8.1, 8.2)

Handouts:

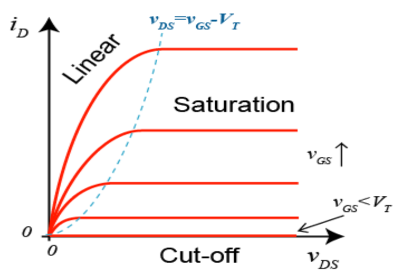
Lecture 22 notes

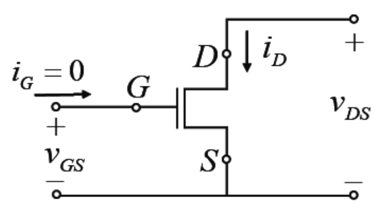
Announcements:

This lecture is being recorded and it will be posted in the certificates-protected part of the 6.002 website

1. Review of MOSFET

- MOSFET output characteristics





- *Cut-off*: $v_{GS} \leq V_T$

$$i_D = 0$$

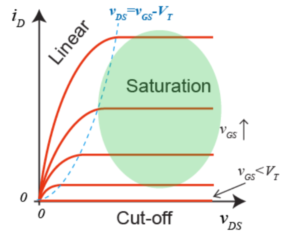
- *Linear* or *triode*: $v_{GS} > V_T$, $v_{DS} \leq v_{GS} - V_T$

$$i_D = K(v_{GS} - V_T - \frac{v_{DS}}{2})v_{DS}$$

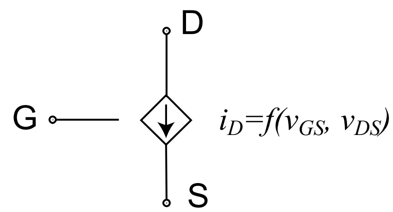
- *Saturation*: $v_{GS} > V_T$, $v_{DS} \geq v_{GS} - V_T$

$$i_D = \frac{K}{2}(v_{GS} - V_T)^2$$

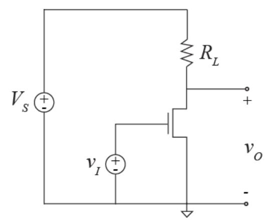
- MOSFET model for amplification:



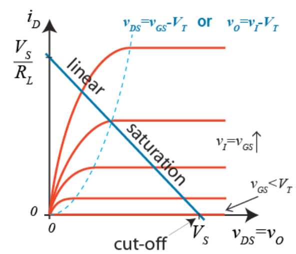
- Equivalent circuit model:



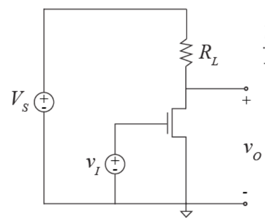
2. MOSFET amplifier (cont.)

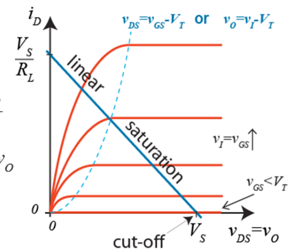


Load diagram:

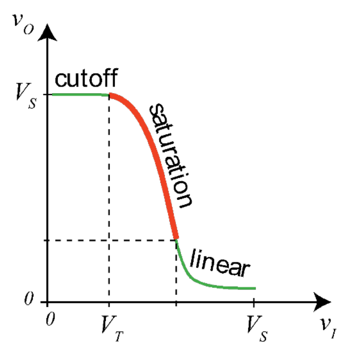


For proper operation, MOSFET must be biased in saturation.



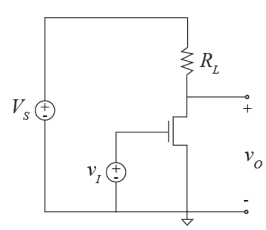


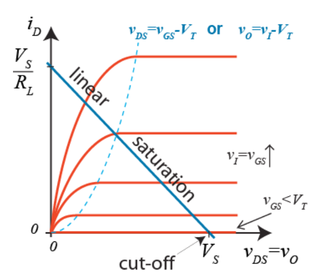
- Transfer characteristics:

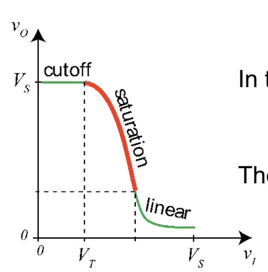


$$v_O = V_S - \frac{KR_L}{2}(v_I - V_T)^2$$

- Limits to MOSFET saturation:







MOSFET enters cut-off regime when:

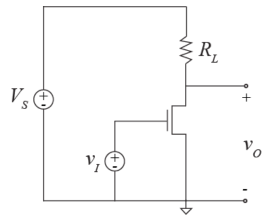
$$v_{GS} \leq V_T$$

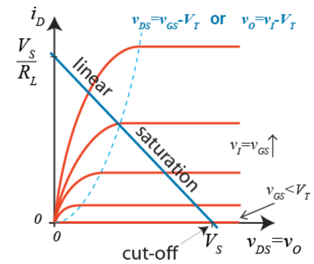
In terms of amplifier variables:

$$v_I \leq V_T$$

The corresponding output is:

$$v_O = V_S$$



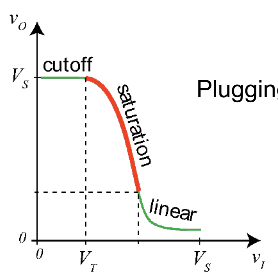


MOSFET enters linear regime when:

$$v_{DS} \le v_{GS} - V_T$$

In terms of amplifier variables:

$$v_O \le v_I - V_T$$



Plugging this into the transfer function:

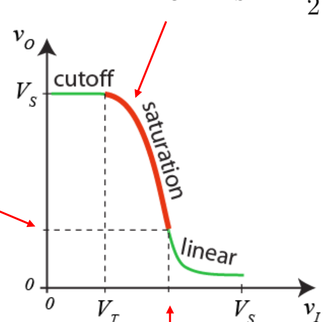
$$v_O = V_S - \frac{KR_L}{2}(v_I - V_T)^2$$

$$v_O \le \frac{\sqrt{1 + 2V_S K R_L} - 1}{KR_L}$$

- In summary:

$$v_O = V_S - \frac{KR_L}{2}(v_I - V_T)^2$$

$$\frac{\sqrt{1 + 2V_S K R_L} - 1}{KR_L}$$



$$V_T + \frac{\sqrt{1 + 2V_S K R_L} - 1}{KR_L}$$

→ Limit to range of acceptable v_I

→ Limit to magnitude of v_O

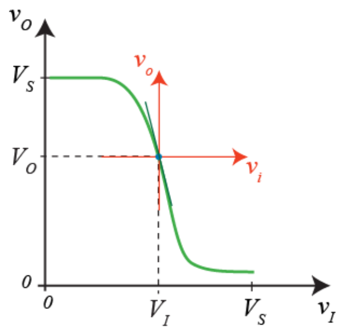
3. Small-signal analysis

- Transfer characteristics of MOSFET amplifier are not linear:

$$v_O = V_S - \frac{KR_L}{2}(v_I - V_T)^2$$

→ distortion!

- Key insight: if magnitude of signal is *small* relative to V_S, transfer characteristics around bias point look fairly *linear* → low distortion.



- If one can linearize, what are the v_o-v_i small-signal transfer characteristics?

- Large-signal transfer characteristics:

$$v_O = V_S - \frac{KR_L}{2}(v_I - V_T)^2$$

- Input is of form:

$$v_I = V_I + v_i$$

- Output should then be of form:

$$\begin{aligned} v_O &= V_O + v_o = V_S - \frac{KR_L}{2}(V_I + v_i - V_T)^2 \\ &= V_S - \frac{KR_L}{2}[(V_I - V_T)^2 + 2v_i(V_I - V_T) + v_i^2] \\ &= V_S - \frac{KR_L}{2}(V_I - V_T)^2 - KR_L(V_I - V_T)v_i - \frac{KR_L}{2}v_i^2 \end{aligned}$$

- Identify terms:
  - Bias terms:

$$V_O = V_S - \frac{KR_L}{2}(V_I - V_T)^2$$

  - Small-signal terms:

$$v_o = -KR_L(V_I - V_T)v_i - \frac{KR_L}{2}v_i^2$$

- Small-signal terms:

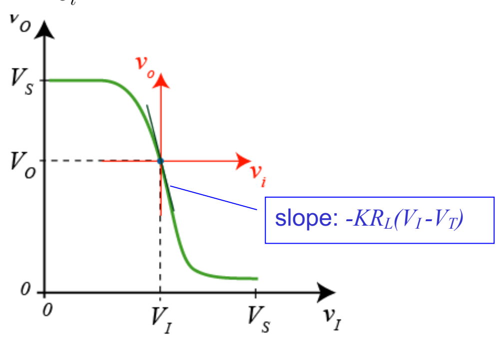
$$v_o = -KR_L(V_I - V_T)v_i - \frac{KR_L}{2}v_i^2$$

- Linearizing means keeping only the linear term:

$$v_o \simeq -KR_L(V_I - V_T)v_i$$

- Small-signal gain:

$$A_v = \frac{v_o}{v_i} \simeq -KR_L(V_I - V_T)$$



$$v_o = -KR_L(V_I - V_T)v_i - \frac{KR_L}{2}v_i^2$$

$$v_o \simeq -KR_L(V_I - V_T)v_i$$

- When is this approximation good?

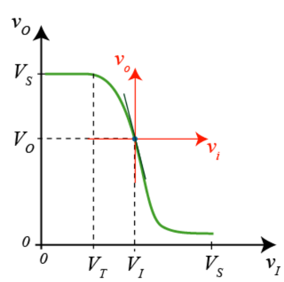
quadratic term << linear term

$$\frac{KR_L}{2}v_i^2 \ll KR_L(V_I - V_T)v_i$$

Or:

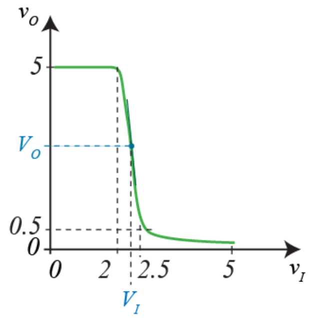
$$v_i \ll 2(V_I - V_T)$$

- The higher the bias, the easier it is to deliver this condition.



- Let's put some numbers: use 2N7000 (V_T=1.8 V, K=0.1 A/V²) with R_L=1 kΩ and V_S=5 V.

Transfer characteristics look like:



If select V_O=2.6 V, then V_I=2.2 V, and:

$$\left|\frac{v_o}{v_i}\right| \simeq -KR_L(V_I - V_T) \sim 40$$

This is true as long as:

$$v_i \ll 2(V_I - V_T)$$

- More generally, obtain small-signal transfer characteristics by **Taylor series** expansion of large-signal transfer characteristics.
- If large-signal transfer characteristics are:

$$v_O = f(v_I)$$

- Then, if v_I=V_I+v_i, expand around (V_I,V_O):

$$v_O = V_O + v_o = f(V_I + v_i) \simeq f(V_I) + \frac{df}{dv_I}|_{V_I} v_i$$

- Small-signal transfer characteristics are:

$$v_o = \frac{df}{dv_I}|_{V_I} v_i$$

- And small-signal gain is:

$$\frac{v_o}{v_i} = \frac{df}{dv_I}|_{V_I}$$

[check that from here, you get the same result as above]

4. Amplifier biasing

- To have signal entirely within acceptable v_I range, apply signal, v_i, on top of *bias,* V_I:

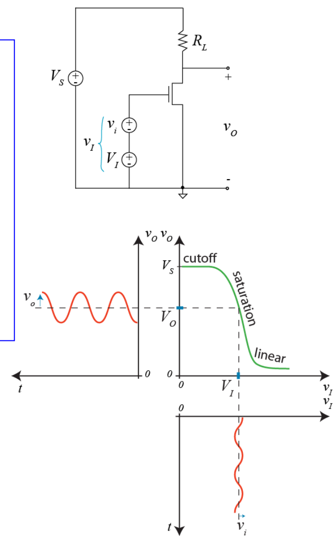
Remember notation:

$v_I(t)=V_I+v_i(t)$

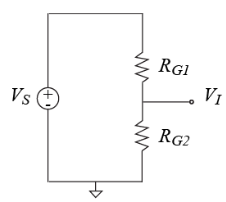
total, bias, small signal

$v_i(t)=V_i \sin\omega t$

amplitude

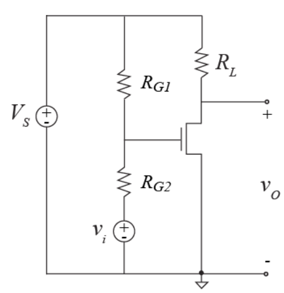


- Problem: need two power supplies, V_S and V_I
- But, can produce V_I out of V_S using voltage divider!



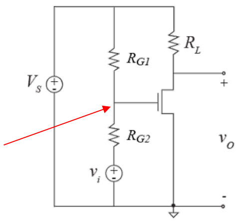
$$V_I = V_S \frac{R_{G2}}{R_{G1}+R_{G2}}$$

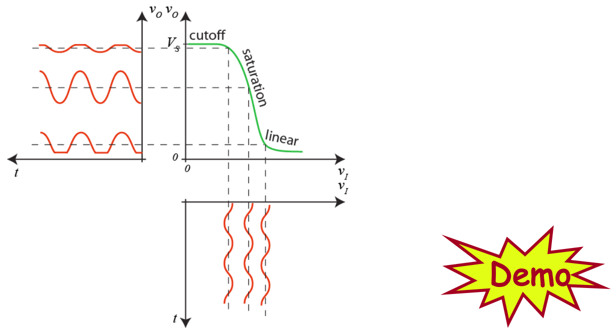
- And connecting everything to the circuit:



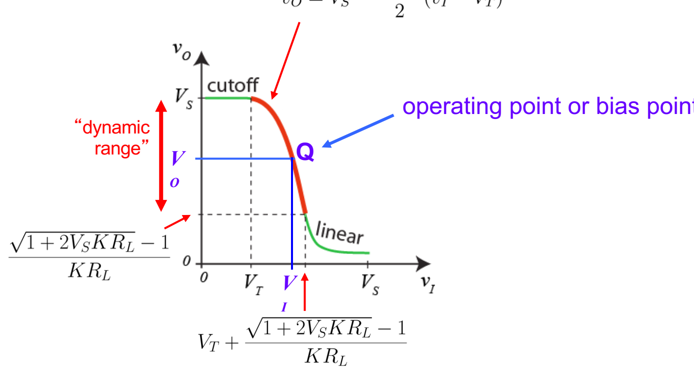
- Resistor values have to be designed correctly to bias MOSFET in saturation

$$V_G = V_S \frac{R_{G2}}{R_{G1}+R_{G2}}$$





- What is a good choice for operating point?
  → Close to middle of the range of acceptable output!



Notice: non-linear transfer characteristics
→ middle of the range of v_O does not correspond to middle of range of v_I

Summary

- To ensure MOSFET operation in the saturation regime, apply bias to the input signal.
- Even if transfer characteristics are non linear, adequate amplifier operation can be obtained for signals of small amplitude.
- For small signals, calculate amplifier gain by linearizing large-signal transfer characteristics of amplifier at bias point.
- In general, this is done by selecting linear term in Taylor series expansion of transfer characteristics around bias point.