6.002 CIRCUITS AND ELECTRONICS

Lecture 19 - **Non-linear components, small-signal analysis**

April 23rd, 2020

Contents:

1. Incremental analysis

Reading Assignment:

Agarwal and Lang, Ch. 4, § § 4.5, 4.6

Handouts:

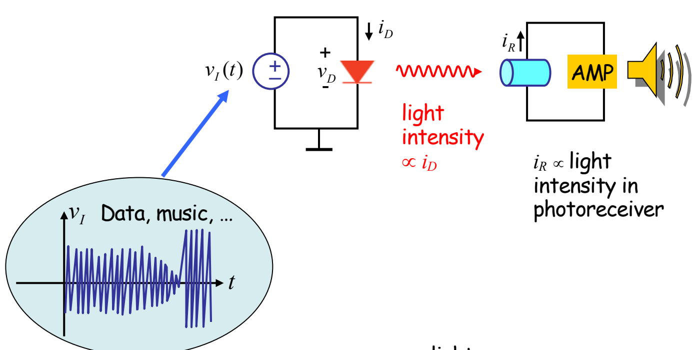
Lecture 19 notes

Announcements:

This lecture is being recorded and will be posted in the certificates-protected part of the 6.002 website

1. Incremental analysis

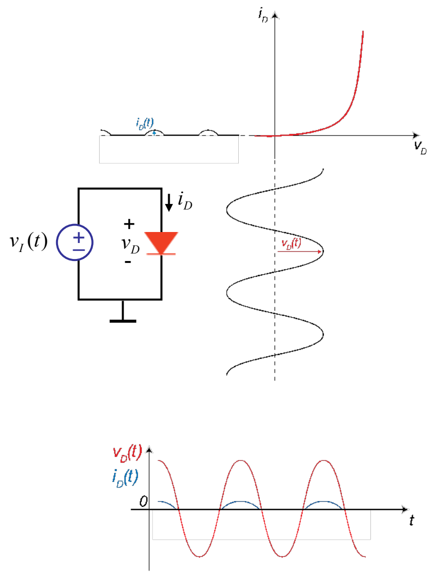
- Consider an optical transmission system



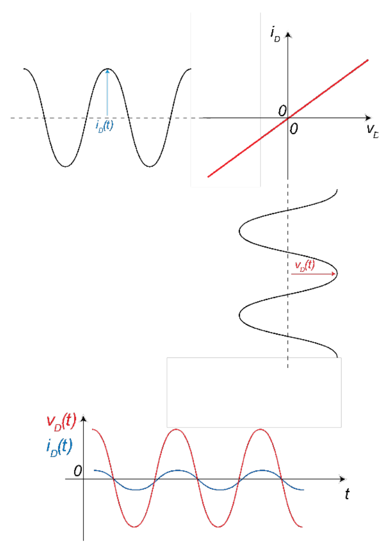
light

$v_I(t) \rightarrow i_D(t)$ ~~~> $i_R(t) \rightarrow$ sound

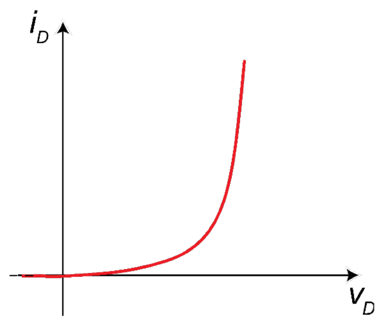
- Problem: LED is non linear → distortion!

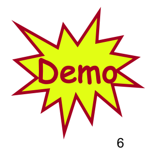


- What we really want is linear characteristics for the LED!

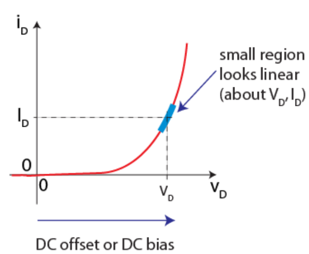


How can we use a non-linear device as a linear one?

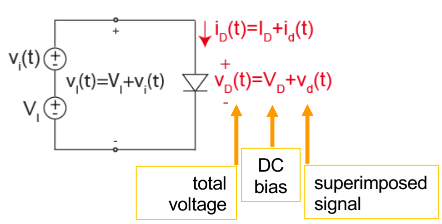


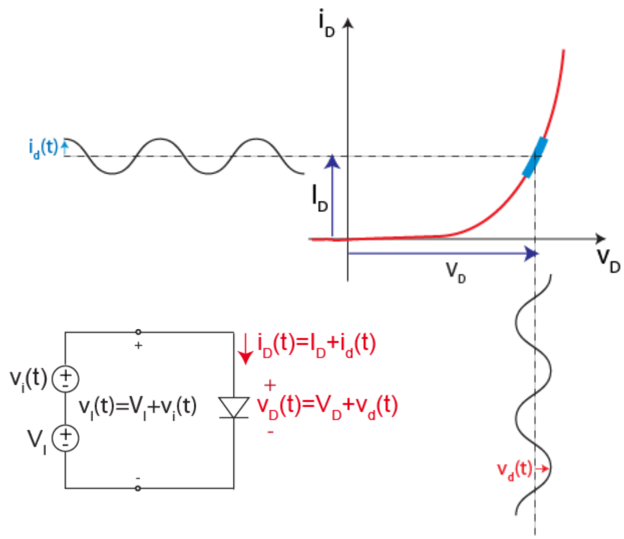


- Insight: small region in i-v characteristics looks linear!



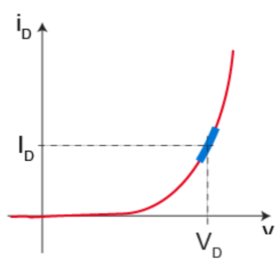
- Apply signal on top of *DC offset* or *DC bias*:



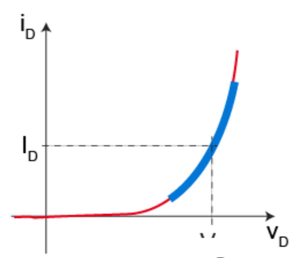


- If $v_d(t)$ is small enough, $i_d(t)$ can be close replica of $v_d(t)$

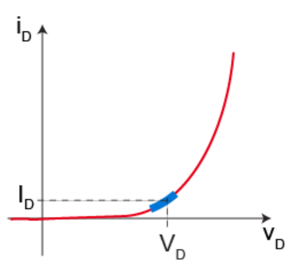
- Consider four possible situations:



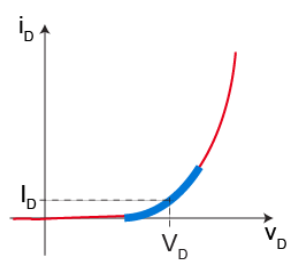
high bias, small signal -> low distortion



high bias, large signal -> high distortion



low bias, small signal -> low output

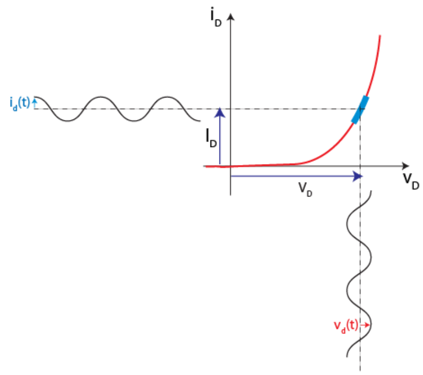


low bias, large signal -> high distortion

Incremental analysis or small-signal method

- Interested in situations where signal has small magnitude:

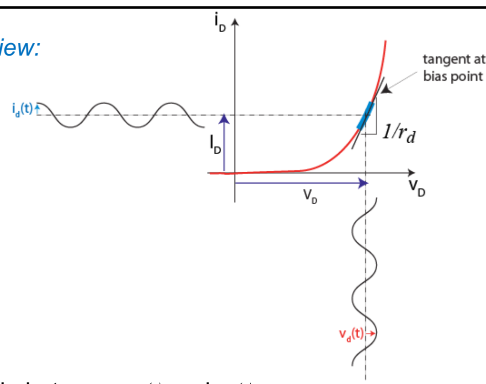
→ want to know $i_d(t)$ in response to $v_d(t)$



- Key insight: if $v_d(t)$ is small enough, device behavior is roughly linear

→ linearize device i-v characteristics

1. The graphical view:



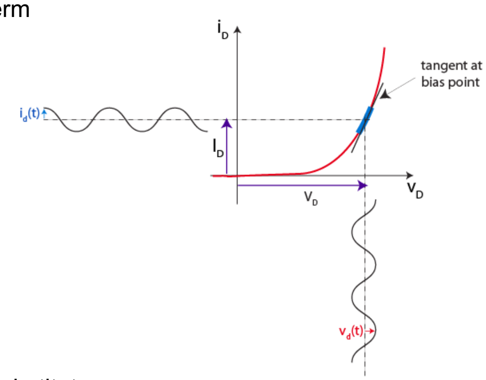
- Relationship between $v_d(t)$ and $i_d(t)$:

$$i_d(t) = \frac{1}{r_d} v_d(t)$$

With $r_d \equiv$ dynamic resistance:

$$r_d(V_D) = \frac{1}{\frac{df(v_D)}{dv_D}\Big|_{v_D=V_D}}$$

2. The mathematical description: do Taylor series expansion and select linear term



- In general terms, substitute:

$$i_D = f(v_D) = f(V_D + v_d) = I_S(e^{qv_D/kT} - 1)$$

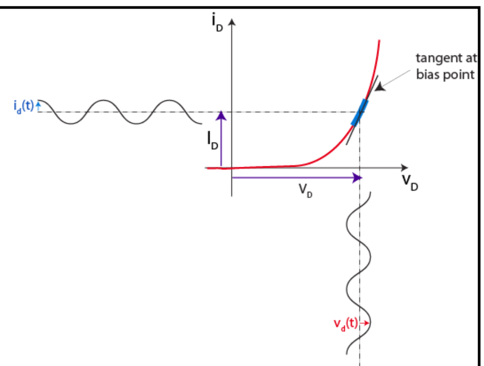
- First three terms of Taylor series expansion:

$$i_D = f(V_D) + \frac{df(v_D)}{dv_D}|_{v_D=V_D} \cdot v_d + \frac{1}{2}\frac{d^2 f(v_D)}{dv_D^2}|_{v_D=V_D} v_d^2 + \cdots$$

- Then:

$$i_D = I_D + i_d = f(v_D) = f(V_D + v_d)$$

$$i_D \simeq f(V_D) + \frac{df(v_D)}{dv_D}|_{v_D=V_D} \cdot v_d$$



- Identify terms:
  - Bias (or operating point):

$$I_D = f(V_D)$$

  - Small signal:

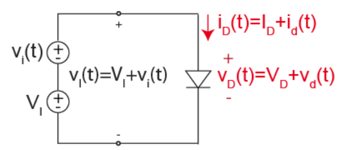
$$i_d(t) = \frac{df(v_D)}{dv_D}|_{v_D=V_D} \cdot v_d(t)$$

linear relation between $v_d(t)$ and $i_d(t)$.

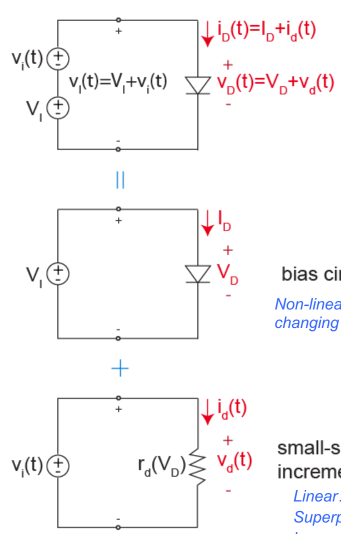
- Proportionality constant between $v_d(t)$ and $i_d(t)$ has units of inverse resistance.

$$r_d(V_D) = \frac{1}{\frac{df(v_D)}{dv_D}|_{v_D=V_D}}$$

3. The circuit view: Small-signal technique effectively breaks problem into two simpler problems:



3. The circuit view: Small-signal technique effectively breaks problem into two simpler problems:



bias circuit

Non-linear (but nothing is changing with time…)

small-signal or incremental circuit

Linear… we can use Superposition, homogeneity, Thevenin and Norton

Example

Consider diode with I_S=1 pA biased at V_D=0.6 V at room temperature. A signal with 1 mV amplitude is applied. What is the amplitude of the small-signal current?

Example

Consider diode with I_S=1 pA biased at V_D=0.6 V at room temperature. A signal with 1 mV amplitude is applied. What is the amplitude of the small-signal current?

Remember i-v characteristics of diode:

$$i_D = I_S(e^{qv_D/kT} - 1)$$

Dynamic resistance of diode:

$$r_d = \frac{1}{\frac{df(v_D)}{dv_D}|_{v_D=V_D}} = \frac{1}{I_S \frac{q}{kT} e^{qv_D/kT}|_{v_D=V_D}}$$

This can be simplified if we remember that, at bias point:

$$I_D = I_S(e^{qV_D/kT} - 1)$$

Then:

$$r_d = \frac{kT}{q(I_D + I_S)}$$

Compute I_D:

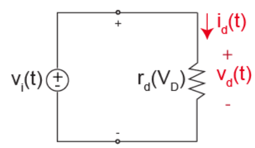
$$I_D \simeq 10^{-12} e^{0.6/0.026} = 11\ mA$$

Dynamic resistance:

$$r_d = \frac{kT}{q(I_D + I_S)} \simeq \frac{kT}{qI_D} = \frac{0.026}{0.011} = 2.4\ \Omega$$

Small-signal current amplitude:

$$i_d = \frac{v_d}{r_d} = \frac{0.001}{2.4} = 0.42\ mA$$



- How "small" does the small signal need to be?

Analysis good if in Taylor series expansion:

Quadratic term << Linear term

$$\frac{1}{2}\frac{d^2 f(v_D)}{dv_D^2}\Big|_{v_D=V_D} v_d^2 \ll \frac{df(v_D)}{dv_D}\Big|_{v_D=V_D} \cdot v_d$$

$$\frac{1}{2} I_S (\frac{kT}{q})^2 e^{qv_D/kT} \cdot v_d^2 \ll I_S (\frac{kT}{q}) e^{qv_D/kT} \cdot v_d$$

Or:

$$v_d \ll 2\frac{kT}{q}$$

At room temperature, this implies:

$$v_d \ll 2 \times 0.026 = 0.052\ V$$

Summary

- Incremental or small-signal analysis: powerful technique suitable for situations with relatively "small" signals.
- Incremental or small-signal model: fully linearized model suitable to solve the small signal problem; can use many circuit solving techniques.
- Values of elements in small-signal model generally depend on the bias point.