6.002 CIRCUITS AND ELECTRONICS

Lecture 8 - **Pulse and impulse response of RC networks**

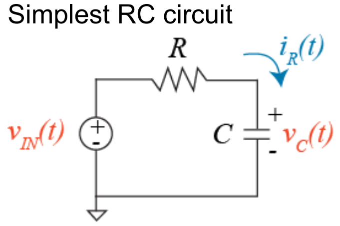
March 3, 2020

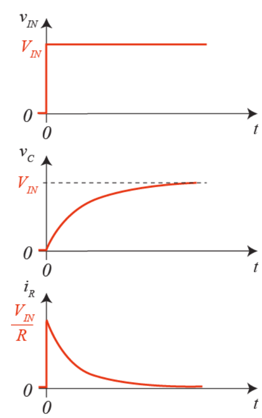
Contents:
1. Review of RC circuits
2. Pulse and impulse response of RC networks

Reading Assignment:
Agarwal and Lang, Ch. 10 (§ § 10.3, 10.4, 10.6.3, 10.6.4)

Handouts:
Lecture 8 notes

- Simplest RC circuit





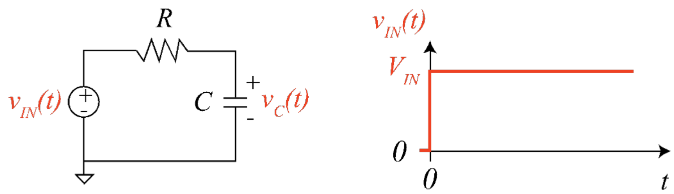
- This is a very important/general circuit as its left side resembles a Thevenin circuit

- Features of solution:
  - Increase of $v_C(t)$ slows down with time
  - Given enough time, $v_C(t)$ saturates
  - $i_R(t)$ peaks at $t=0^+$ and drops from there to eventually zero
  - $v_C(t)$ must be continuous, otherwise infinite current…

- What is the detailed shape? How long does it take for steady-state to be achieved? → need to solve problem mathematically

Mathematical solution (18.03 to the rescue!)

- Given:

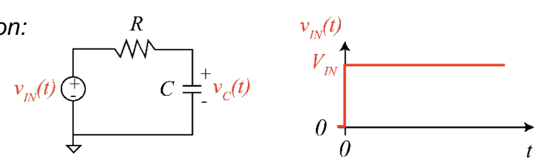


$v_C(t=0^-)=0$

- Want $v_C(t)$

- Steps:
  1. Formulate differential equation
  2. Find particular solution
  3. Find homogeneous solution
  4. Apply initial conditions

1. *Formulate differential equation:*



$v_C(t=0^-)=0$

One node analysis.
Node equation for $v_C(t)$:

$$\frac{v_C(t)-v_{IN}(t)}{R}+C\frac{dv_C(t)}{dt}=0$$

Rearrange:

$$RC\frac{dv_C(t)}{dt}+v_C(t)=v_{IN}(t)$$

Solution of form:

$$v_C(t)=v_{CH}(t)+v_{CP}(t)$$

Homogeneous solution ("natural response")

Particular solution ("forced response")

2. *Find particular solution:*

Interested in solution for $t>0$. In this range:

$$v_{IN}(t) = V_{IN}$$

Differential equation becomes:

$$RC\frac{dv_C(t)}{dt} + v_C(t) = V_{IN}$$

Particular solution satisfies differential equation:

$$RC\frac{dv_{CP}(t)}{dt} + v_{CP}(t) = V_{IN}$$

Try solution of form:

$$v_{CP}(t) = K$$

Then:

$$v_{CP}(t) = V_{IN}$$

3. Find homogeneous solution

Homogeneous differential equation:

$$RC\frac{dv_{CH}(t)}{dt} + v_{CH}(t) = 0$$

Solution of the form:

$$v_{CH}(t) = Ae^{-\frac{t}{\tau}}$$

Value of τ that satisfies differential equation:

$$\tau = RC$$

Then:

$$v_{CH}(t) = Ae^{-\frac{t}{RC}}$$

4. Apply initial conditions

Total solution so far:

$$v_C(t) = V_{IN} + Ae^{-\frac{t}{RC}}$$

Initial condition: $$v_C(0) = 0$$

[voltage across capacitor can't change abruptly]

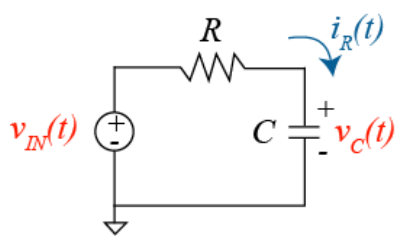
Then:

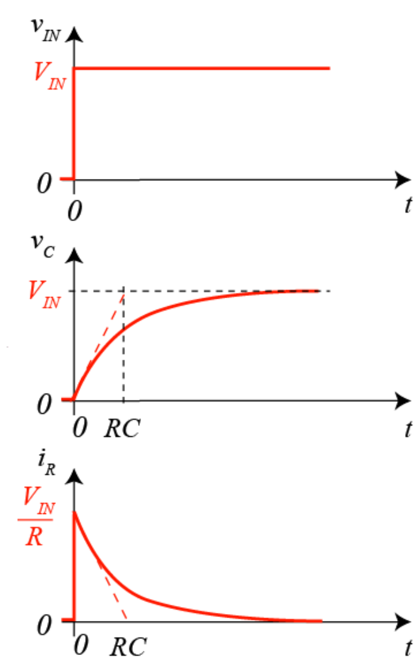
$$A = -V_{IN}$$

Final solution:

$$v_C(t) = V_{IN}(1 - e^{-\frac{t}{RC}})$$

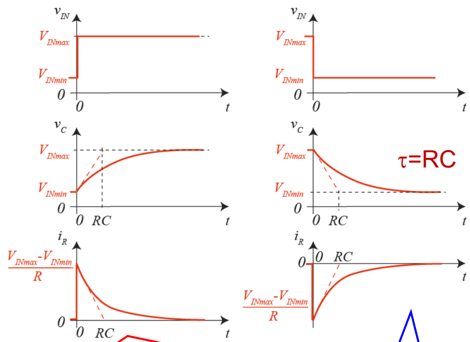
- Can also compute current through R:





$$i_R(t) = \frac{v_{IN}(t) - v_C(t)}{R} = \frac{V_{IN}}{R}e^{-\frac{t}{RC}}$$

More generally:



$$v_C(t) = V_{INmin} + (V_{INmax} - V_{INmin})(1 - e^{-\frac{t}{RC}})$$

$$i_R(t) = \frac{V_{INmax}-V_{INmin}}{R} e^{-\frac{t}{RC}}$$

$$v_C(t) = V_{INmin} + (V_{INmax} - V_{INmin})e^{-\frac{t}{RC}}$$

$$i_R(t) = -\frac{V_{INmax}-V_{INmin}}{R} e^{-\frac{t}{RC}}$$

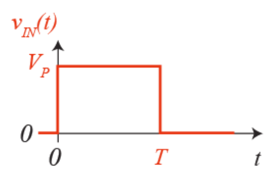
1. Voltage across a capacitor is always continuous (unless infinite current)

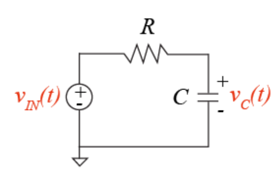
2. If voltage across capacitor doesn't change… the current is 0… Capacitor behaves like an open circuit

3. Currents/voltages at the extremes (i.e. t=0⁻ or infinity) can typically be calculated from initial conditions + 1. and/or 2 .above

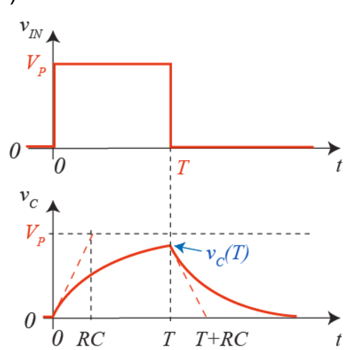
2. Pulse and impulse response of RC networks

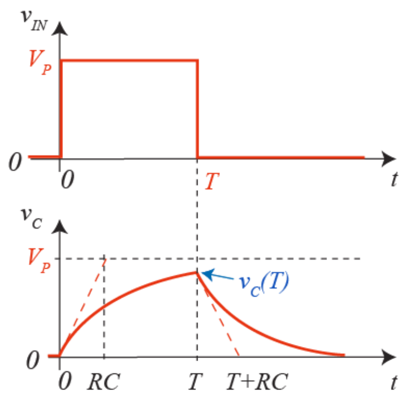
- Apply *pulse* to RC circuit:





Response ($v_C(0)=0$):





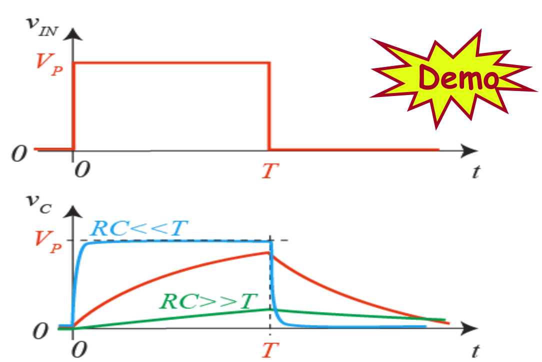
Mathematically (by inspection…):

$$v_C(t) = V_P(1 - e^{-\frac{t}{RC}}) \quad \text{for} \quad 0 \leq t \leq T$$

$$v_C(t) = V_P(1 - e^{-\frac{T}{RC}})e^{-\frac{t-T}{RC}} \quad \text{for} \quad T \leq t$$

Limits:

– Long pulse: $RC<<T$
– Short pulse: $RC>>T$



Short-pulse limit: Taylor-series expand response from $0 \leq t \leq T$:

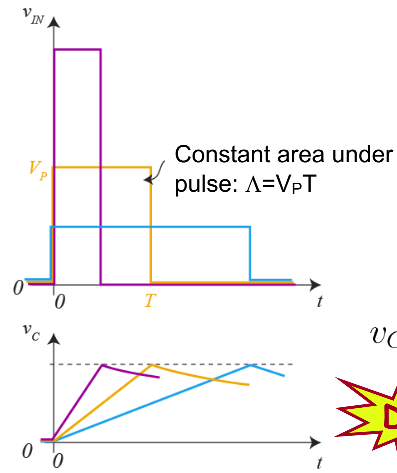
$$v_C(t) = V_P(1 - e^{-\frac{t}{RC}}) \simeq V_P[1 - (1 - \frac{t}{RC})] = \frac{V_P}{RC}t$$

Capacitor voltage at end of pulse:

$$v_C(T) = \frac{V_P}{RC}T$$

Interesting! $v_c(T)$ depends on the "area" under the pulse!

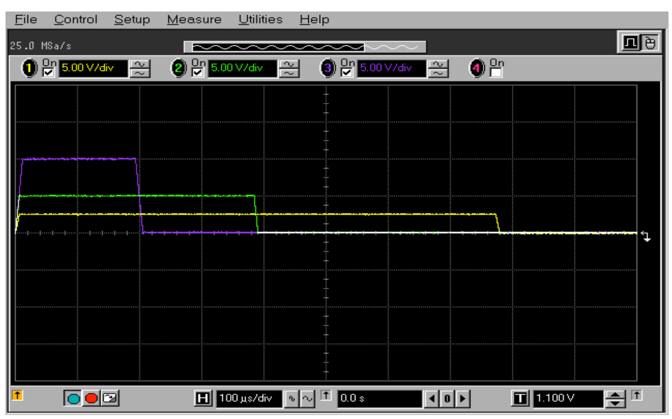
- In short pulse limit, consider scaling down pulse width keeping pulse area constant:



$$v_C(T) = \frac{\Lambda}{RC}$$

The limit of a infinitely short pulse with finite area is called *impulse*

- In short pulse limit, consider scaling down pulse width keeping pulse area constant:



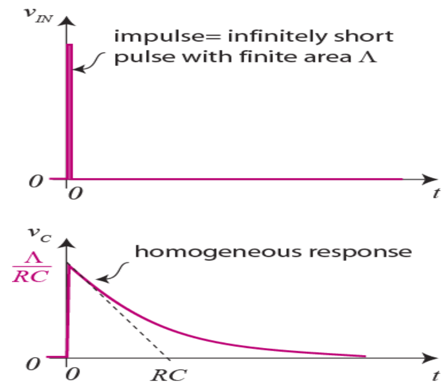
The limit of a infinitely short pulse with finite area is called *impulse*

Impulse response

Impulse: infinitely short pulse with finite area

Why is it important?

→Impulse response is homogeneous response, independent of details of pulse (scales with area)



Homogeneous response=natural response → effective way to learn about a system

Summary

- RC effects in logic circuits dominate dynamic behavior
- Short-pulse limit of RC network: linear behavior → capacitor charge and voltage at end of pulse depends on product of pulse height and width
- Limit of very short pulse with finite area under the pulse: impulse response
- Impulse response important because it is the homogeneous response of a system