MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

QUIZ 2 - 6.002 Circuits and Electronics

April 23, 2019

Total Points: 120 **Time Limit: 120 minutes**

YOUR NAME____

RECITATION: 11 am 12 pm 1 pm

General Instructions:

1. Please do all of your work in the spaces provided in this examination booklet. Place your answer for each question in the space provided on this booklet.

2. The exam consists of 4 problems on pages 2-21. Please make sure you have all of the pages. Use the space immediately following each question to show your work and the answer to the question. If you need more space, use the back of the preceding page.

3. All sketches must be adequately labeled

4. You will be graded on both your solution (that is, the work shown) and your final answer. It is possible to get the right answer, but not receive full credit if your reasoning is unclear. A few words of explanation are required.

5. Indicate units on all numerical answers.

6. The exam is closed books but calculators and a single two-sided page of notes are allowed.

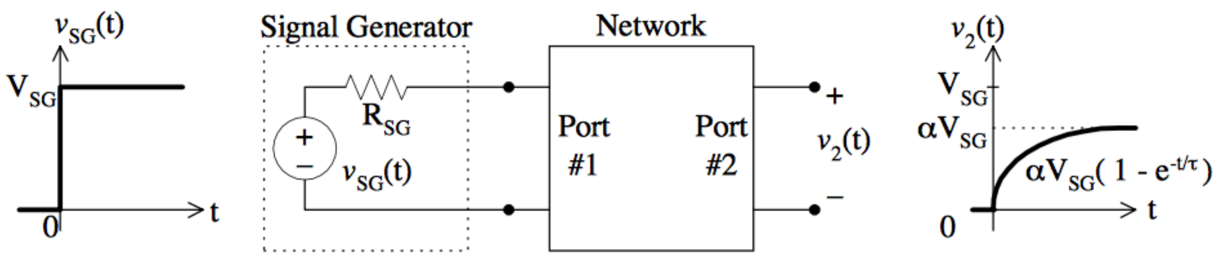
7. Please do not remove any pages from this exam booklet.

Grade:	**Problem 1:**	**(/20)**
	Problem 2:	**(/30)**
	Problem 3:	**(/30)**
	Problem 4:	**(/40)**

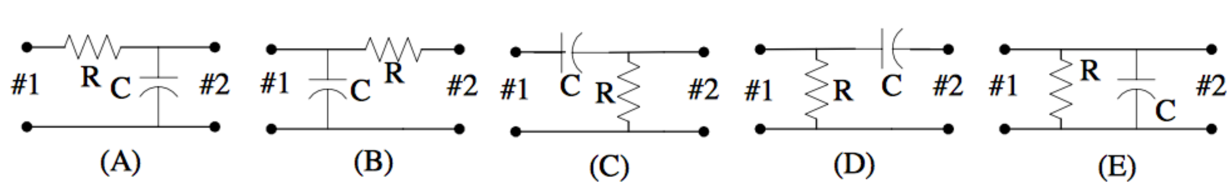
Total grade:

Problem 1 [20 points]: Transients in RL and RC Networks

A signal generator having Thevenin resistance R_{SG} is connected to Port #1 of a two-port network as shown below. Assume that the Thevenin voltage of the signal generator is zero for a very long time prior to t = 0. At $t = 0$, the Thevenin voltage $v_{SG}(t)$ of the signal generator takes a step from zero to V_{SG}, and the voltage $v_2(t)$ is measured at Port #2. Note that α is a unitless constant satisfying 0< α <1, and τ is a time constant.

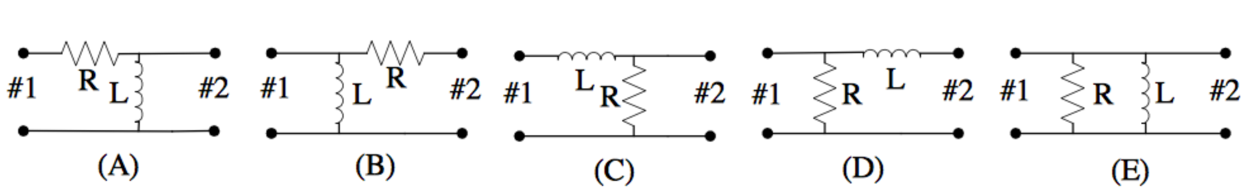


a) [5 points] Which of the following could be the two-port network?



Network (Circle One):	A	B	C	D	E

b) [5 points] Which of the following could be the two-port network?



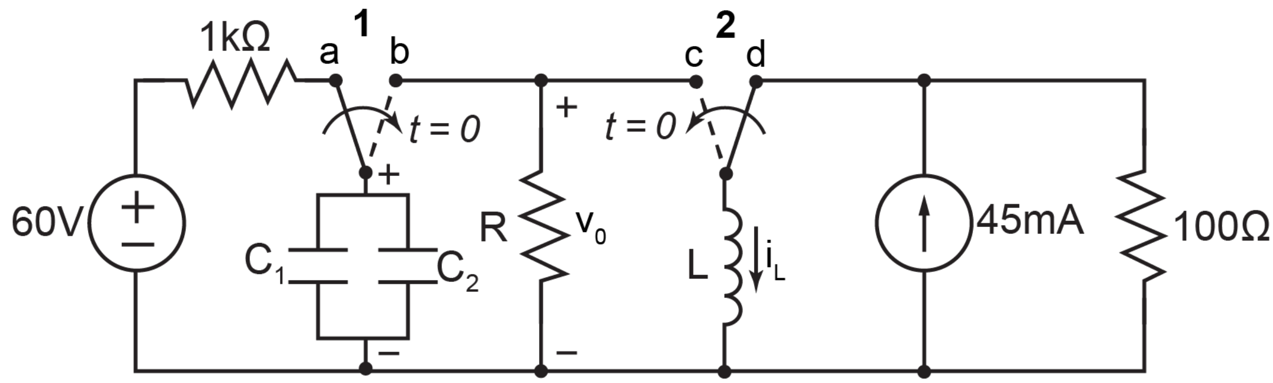
Network (Circle One):	A	B	C	D	E

c) [10 points] Determine the values of R and L in the network you chose in Part b). Express the values in terms of V_{SG}, R_{SG}, α and τ.

R=	L=

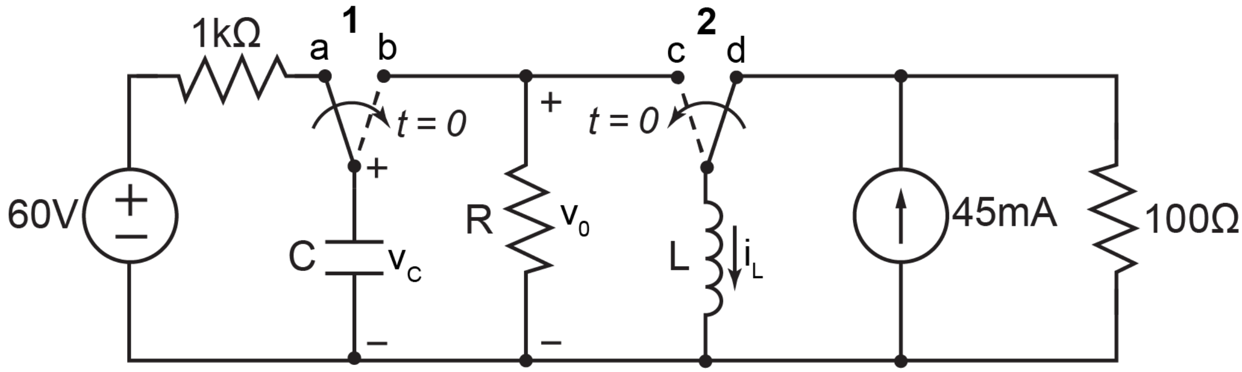
Problem 2 [30 points]: RLC Circuits

Consider the circuit shown below. In this circuit, the two switches operate synchronously. When switch 1 is in position **a**, switch 2 is in position **d**. When switch 1 moves to position **b**, switch 2 moves to position **c** simultaneously.



a) [2 points] This circuit can be simplified by replacing C_1 and C_2 by their equivalent capacitance. Write an expression for this equivalent capacitance (C) in terms of C_1 and C_2.

Using the equivalent capacitance, the network can now be simplified as shown below.



Consider that switch 1 has been in position **a** for a long time. At t = 0, the switches move to their alternate positions. Assuming equivalent capacitance C = 62.5 nF, R = 1.6 kΩ and L = 1 H, find an expression for $v_0(t)$ for $t \geq 0$. To do so, follow the steps below.

b) [4 points] Derive a second-order differential equation that describes the evolution of v_0. Write this in terms of v_0, R, C, and L.

Differential Equation	

c) [6 points] Determine if the response to this system is going to be underdamped, overdamped, or critically damped. Hint: Find and use values of ω_0 and α to determine the response type.

ω_0	
α	
Overdamped, Underdamped, Or Critically damped?	

d) [3 points] What is $v_c\ (0^+)$? Note that C represents the equivalent capacitor.

$v_c\ (0^+) =$

e) [3 points] What is $i_L(0^+)$? Note that L represents the inductor.

$i_L(0^+) =$

f) [6 points] Now find an expression for v_0(t) for t ≥ 0. All constants should be evaluated using w_0, α, and the initial conditions. Instead of using your answers from parts D and E though, for the initial conditions assume $v_c\ (0^+) = 50\ V$ and $i_L(0^+) = 35\ mA$.

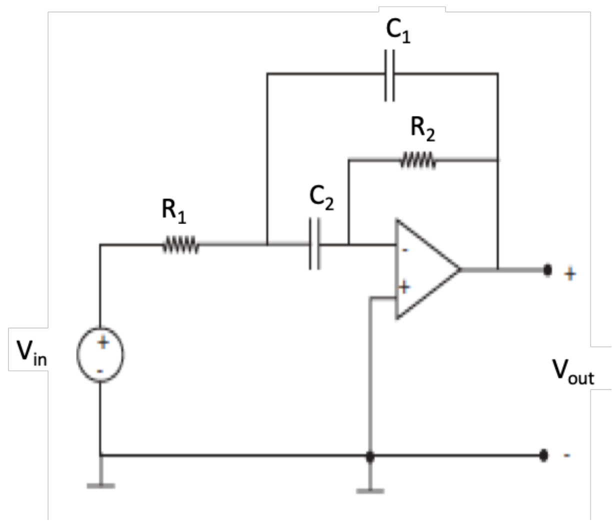
v_0(t)=

g) [6 points] Now consider that we would like this circuit to have a quality factor of 2. Assuming C = 62.5 nF and L = 1 H, what value of R should be used to achieve this quality factor? Is the response overdamped, underdamped, or critically damped?

R	
Overdamped, Underdamped, Or Critically damped?	

Problem 3 [30 points]: Op-Amps with memory elements

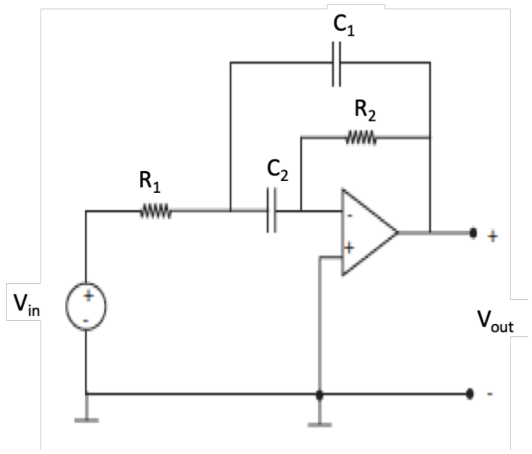
An ideal operational amplifier is connected as shown in the figure below. In this circuit C_1=C_2=0.01 µF, R_1=10 Ω, and R_2 = 1kΩ



a) [5 points] What is the voltage gain of the amplifier (V_{out}/V_{in}) under sinusoidal steady state for ω=0? A numerical value is expected.

b) [5 points] What is the voltage gain of the amplifier (V_{out}/V_{in}) of the amplifier under sinusoidal steady state for ω=∞? A numerical value is expected.

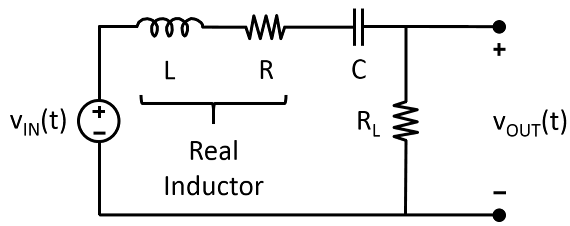
c) [10 points] Find the expression for $V_{out}(j\omega)/V_{in}(j\omega)$ under sinusoidal steady state operation. For your convenience, you can find the circuit diagram below. Please write the expression as a function of R_1, R_2, C_1, C_2, and ω.



d) [10 points] Sketch the magnitude for the expression in c). Indicate the frequency at which the peak occurs (if any). This frequency should be written as a function of R_1, R_2, C_1 and C_2.

Problem 4: RLC Filtering – 40 Points

The circuit diagram of a band-pass filter is shown below. The filter is constructed with a series inductor-capacitor resonator. The proximity card reader studied in lab used a similar resonator for its notch filter. The main difference between the resonator used here and the one used in lab is that the resonator used here has a more realistic model of the inductor which includes a series resistance.



(4A) (4 Points) Let the filter operate in the sinusoidal steady state such that $v_{\rm IN}(t) = V_{\rm in}\cos(\omega t)$, where $V_{\rm in}$ is a real constant, and $v_{\rm OUT}(t) = \Re\left\{\tilde{V}_{\rm out}(\omega)e^{j\omega t}\right\}$, where $\tilde{V}_{\rm out}(\omega)$ is a complex function of the frequency ω. Determine the transfer function $\tilde{V}_{\rm out}(\omega)/V_{\rm in}$ in terms of C, L, R, $R_{\rm L}$ and ω.

$\tilde{V}_{\rm out}(\omega)/V_{\rm in} =$

(4B) (4 Points) When operating in the sinusoidal steady state, the filter output can be expressed in the alternative form $v_{\mathrm{OUT}}(t) = V_{\mathrm{out}}(\omega)\cos(\omega t + \phi(\omega))$ where $V_{\mathrm{out}}(\omega)$ and $\phi(\omega)$ are real functions of the frequency ω. The amplitude $V_{\mathrm{out}}(\omega)$ can further be written in the form

$$V_{\mathrm{out}}(\omega) = G\frac{(\omega\omega_\circ/Q)V_{\mathrm{in}}}{\sqrt{(\omega^2-\omega_\circ^2)^2+(\omega\omega_\circ/Q)^2}} \quad .$$

Determine the constants G, Q, and $\omega_\circ$, in terms of C, L, R, and R_{L}.

G	Q	$\omega_\circ$

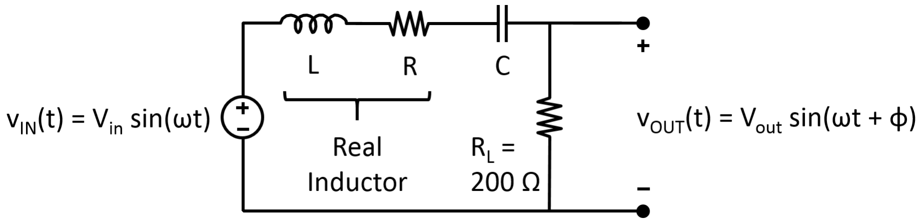
(4C) (4 Points) Again, when operating in the sinusoidal steady state, the filter output can be expressed in the form $v_{\mathrm{OUT}}(t) = V_{\mathrm{out}}(\omega)\cos(\omega t + \phi(\omega))$. Determine the phase $\phi(\omega)$ in terms of C, L, R, R_{L}, and ω.

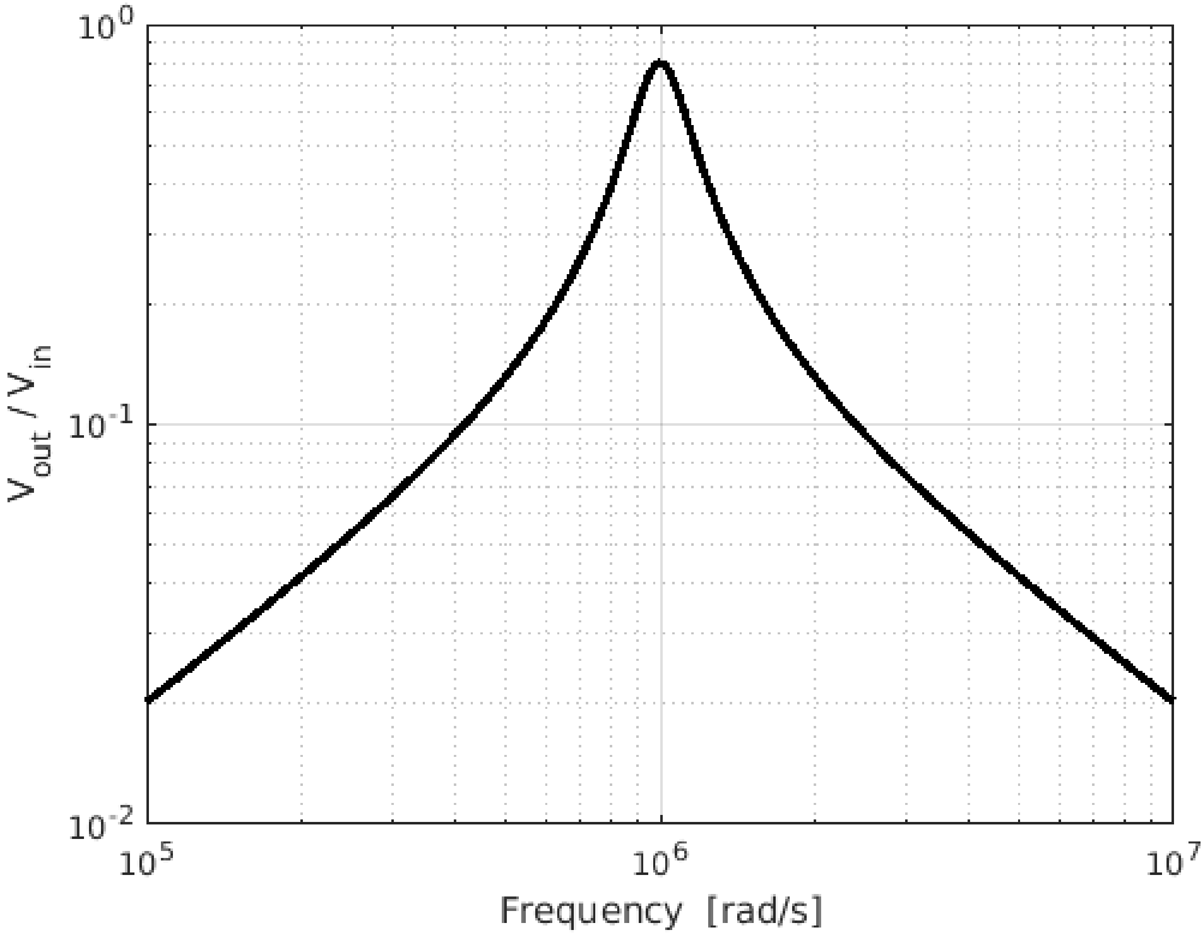
$\phi(\omega) =$

(4D) (4 Points) Derive a differential equation that relates the filter output $v_{\mathrm{OUT}}(t)$ to the filter input $v_{\mathrm{IN}}(t)$. *You need not solve the differential equation.*

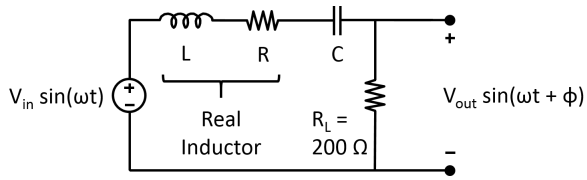
Differential Equation:

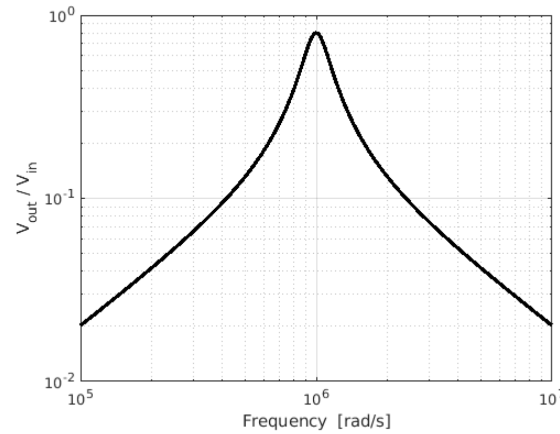
When operating in the sinusoidal steady state with $R_L = 200\ \Omega$, as shown below, the band-pass filter input-output transfer function magnitude, V_{out}/V_{in}, is measured also as shown below. Use the numerical data to answer the questions that follow.





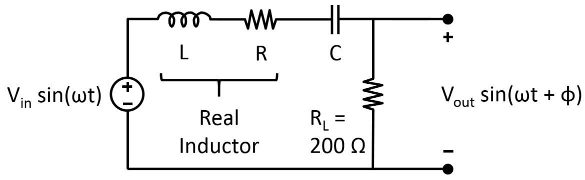
(4E) (4 Points) Based on the numerical transfer function magnitude, determine a numerical value for the resistance R.

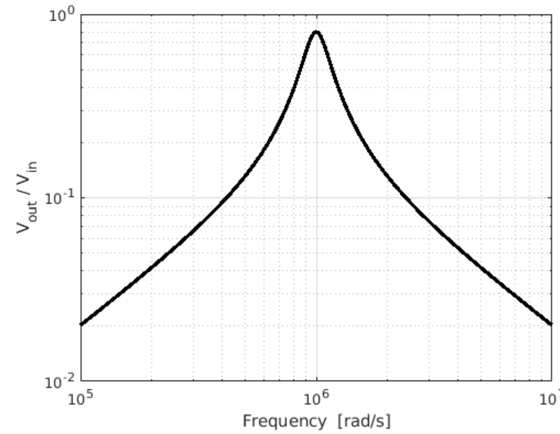




$R =$

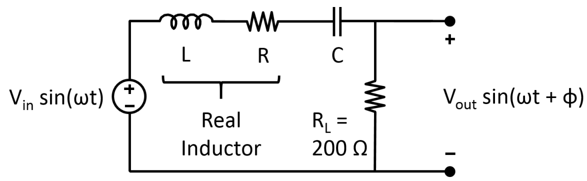
(4F) (4 Points) Based on the numerical transfer function magnitude, determine a numerical value for the quality factor Q.

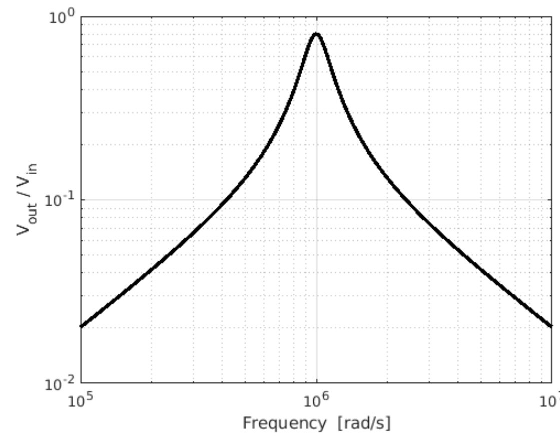




$Q =$

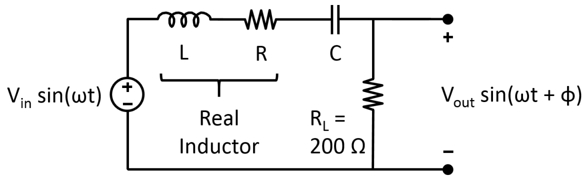
(4G) (8 Points) Based on the numerical transfer function magnitude, determine numerical values for the capacitance C and inductance L.

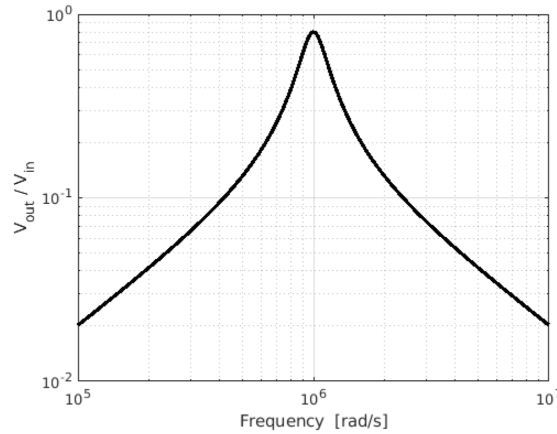




C	L

(4H) (4 Points) Suppose that the input to the filter has been $v_{IN}(t) = V_1 \sin(\omega_1 t)$ for a very long time. Then, at $t = 0$, the input switches to $v_{IN}(t) = V_2 \sin(\omega_2 t)$. Based on the numerical transfer function magnitude, how long will it take before the output approximately settles to a new steady state? Provide both an answer and a brief justification for your answer.





How Long?
Justification

(4I) (2 Points) If R were to increase, would the peak value of the input-output relation $|V_{\text{out}}(\omega)/V_{\text{in}}|$ increase, decrease or remain unchanged?

Increase	Decrease	Unchanged

(4J) (2 Points) If R were to increase, would the frequency at which the peak value of the input-output relation $|V_{\text{out}}(\omega)/V_{\text{in}}|$ occurs increase, decrease or remain unchanged?

Increase	Decrease	Unchanged