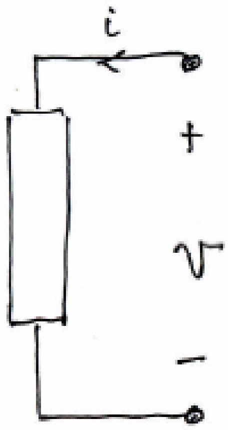
6.002 – Lecture 2

Circuit Analysis Simplifications

- Parallel & Series Reductions
- Node Analysis

Device Laws



Voltage Source $\Rightarrow$ Known v

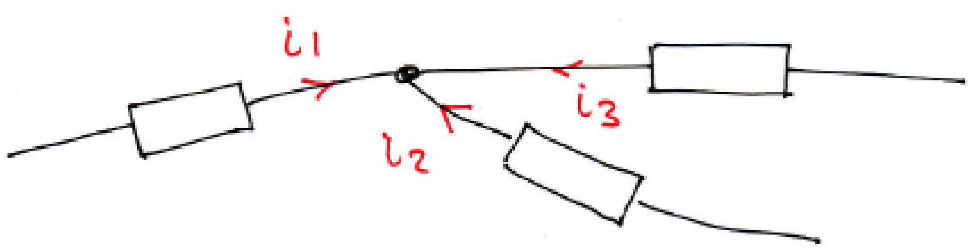
Current Source $\Rightarrow$ Known i

Linear Resistor $\Rightarrow$ $v = Ri$

Power In $= vi$

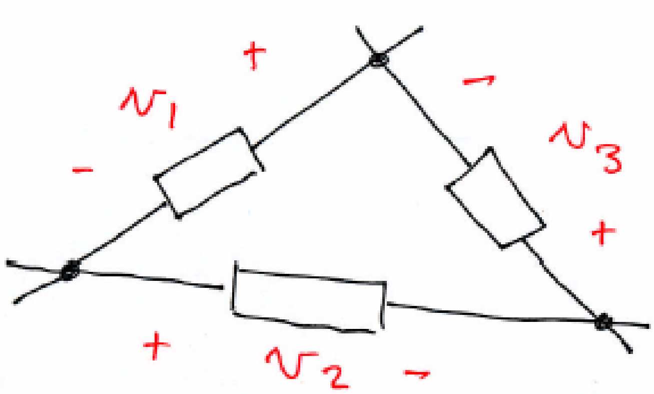
KCL

$$i_1 + i_2 + i_3 = 0$$



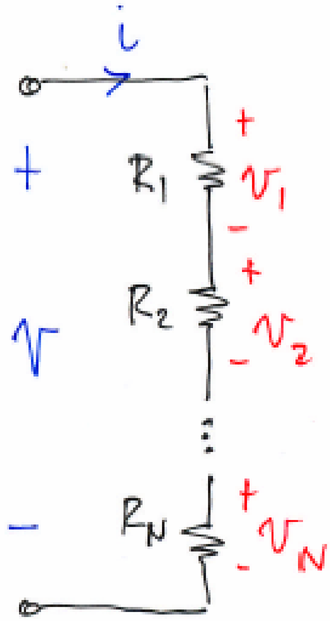
KVL

$$v_1 + v_2 + v_3 = 0$$



Series Resistors

Common Current i

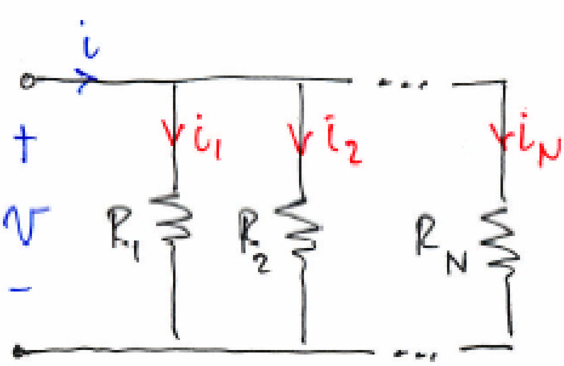


$$
\begin{aligned}
v &= v_1 + v_2 + \dots v_N \\
&= R_1 i + R_2 i + \dots R_N i \\
&= (R_1 + R_2 + \dots R_N)\, i \\
&\equiv R\, i
\end{aligned}
$$

$$R = R_1 + R_2 + \dots R_N$$

Parallel Resistors

Common Voltage v

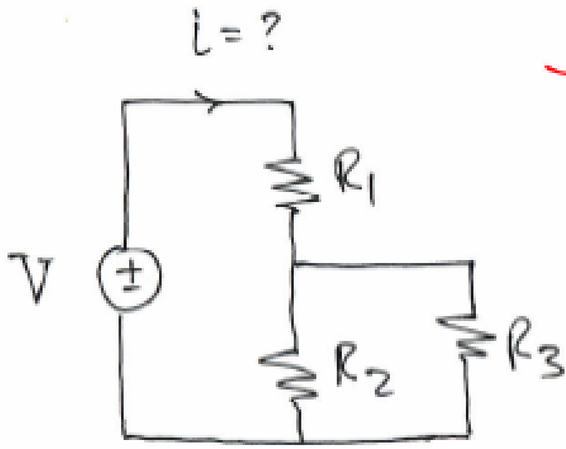


$$
\begin{aligned}
i &= i_1 + i_2 + \dots i_N \\
&= G_1 v + G_2 v + \dots G_N v \\
&= (G_1 + G_2 + \dots G_N)\, v \\
&\equiv G\, v
\end{aligned}
$$

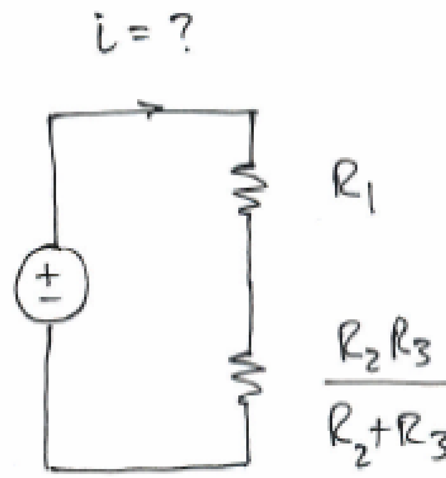
$$G = G_1 + G_2 + \dots G_N$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_N}$$

Simplification Process

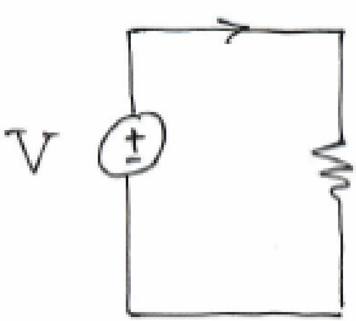


Parallel Reduction



Series Reduction

$i = V/R$



$R_1 + \dfrac{R_2 R_3}{R_2 + R_3} \equiv R$

Use voltage and current dividers, and device laws to determine remaining branch variables.

(Simplified) Node Analysis

(0) Draw the circuit neatly.

(1) Select a reference node from which all other node voltages are measured. Define its voltage to be zero. This node is "ground" (⏚ ⏚ ⊥).

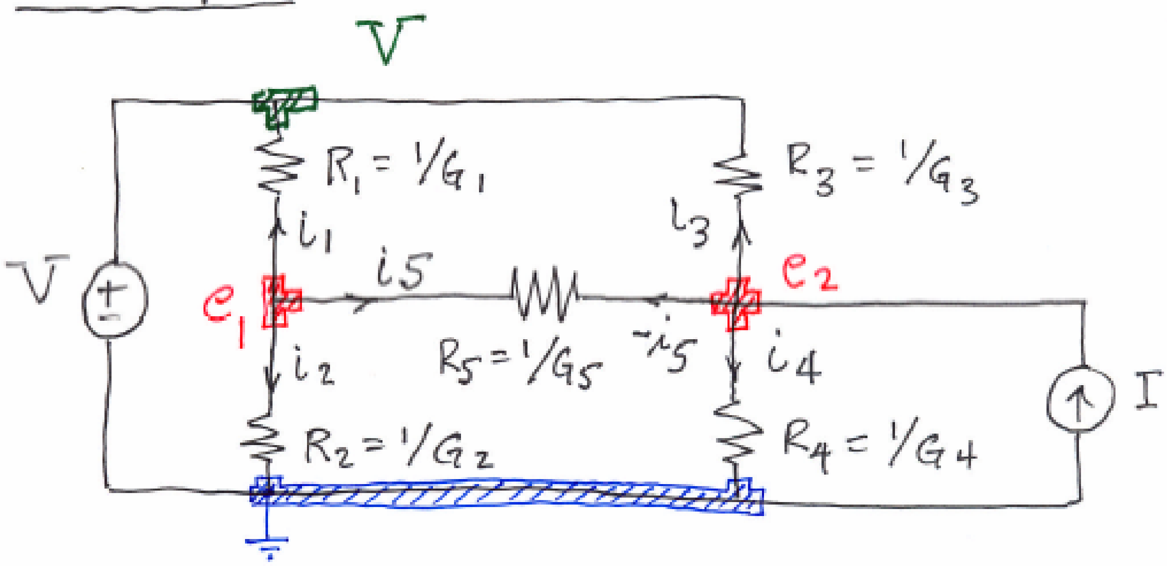
(2) Label all remaining un-sourced node voltages with respect to ground. These are the primary analysis variables.

(3) Write KCL for all nodes with an unknown voltage, and immediately substitute the device laws and KVL. The resulting equations are now in terms of the node voltages.

(4) Solve the KCL equations for the node voltages.

(5) Back solve for branch variables.

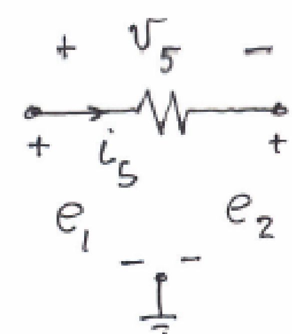
Example



e_1 Node: $i_1 + i_2 + i_5 = 0$

$$G_1(e_1 - V) + G_2(e_1 - 0) + G_5(e_1 - e_2) = 0$$

$$e_1(G_1 + G_2 + G_5) + e_2(-G_5) = V(G_1)$$



$$e_1 = v_5 + e_2$$

$$v_5 = e_1 - e_2$$

e_2 Node: $i_3 + i_4 + (-i_5) - I = 0$

$$G_3(e_2 - V) + G_4(e_2 - 0) + G_5(e_2 - e_1) - I = 0$$

$$e_1(-G_5) + e_2(G_3 + G_4 + G_5) = V(G_3) + I$$

Solution

$$\begin{bmatrix} G_1+G_2+G_5 & -G_5 \\ -G_5 & G_3+G_4+G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix}$$

$$\frac{\begin{bmatrix} G_3+G_4+G_5 & G_5 \\ G_5 & G_1+G_2+G_5 \end{bmatrix}}{(G_1+G_2+G_5)(G_3+G_4+G_5) - G_5^2} \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$e_1 = \frac{(G_3+G_4+G_5)\, G_1 V + G_5 (G_3 V + I)}{G_1G_3 + G_1G_4 + G_1G_5 + G_2G_3 + G_2G_4 + G_2G_5 + G_5G_3 + G_5G_4}$$

$$e_2 = \frac{G_5 G_1 V + (G_1+G_2+G_5)(G_3 V + I)}{(\text{Same Denominator})}$$

Demo

$$\left.\begin{aligned} R_1 &= R_4 = 8.2\,k\Omega \\ R_2 &= R_3 = 3.9\,k\Omega \\ R_5 &= 1.5\,k\Omega \\ V &= 3\,V \\ I &= 0\,A \end{aligned}\right\} \quad \begin{aligned} e_1 &= 1.382\,V \\ e_2 &= 1.618\,V \end{aligned}$$

Warning

The node method as presented does not handle floating voltage sources. Next lecture!