Let's look at the circuit at the left. This is actually called a source follower amplifier (but we don’t need to worry about that too much here). Instead we can just see that there’s a MOSFET and a current source with a constant current of ISS on it.

The voltage we care about finding is the output voltage, vO which has a large and small signal component vO = VO + vo

We’d like to figure out a few things about it based on its input:
vl = VI + vi

and the Bias current ISS (which we’ll note is large signal only because it is just a constant current source).

Let’s first determine what the output bias point will be (the large signal component of vO) as a function of the large signal component of VI (VI) and ISS. Let’s assume that the FET is working in SATURATION mode:

\[ i_D = \frac{K}{2} \left( V_{GS} - V_T \right)^2 \]
\[ V_{GS} = V_I - V_O \]
\[ i_D = I_{SS} \]
\[ I_{SS} = \frac{K}{2} \left( V_I - V_O - V_T \right)^2 \]
\[ V_I - \sqrt{\frac{2 I_{SS}}{K}} - V_T = V_O \]
\[ V_O = V_I - \sqrt{\frac{2 I_{SS}}{K}} - V_T \]

The three starting equations came from our MOSFET in saturation equation (top right), and then two pieces of info we extracted from the circuit. The equation about vGS is coming from KVL and analyzing equivalent voltages in the circuit, and the line about ID is coming from KCL...all current through that MOSFET’s drain-source path is going to be regulated by that current source ISS. The large signal model/bias point of the system is just:

or in other words, our combined equation solved out for when only large signal inputs are present.
Returning to our original equation for $v_O$, for a second, if we were to plot this out you'd get something like the following:

$$v_O = v_I - \sqrt{\frac{2I_{SS}}{W}} - V_T$$

I don't want to obsess too much over this (and we can ignore the outer bounds of this where the transistor leaves saturation mode), but you can see that the input to output relationship looks just like shown...

What we'd like to do now is come up with a small signal model for this system and then incorporate that into our full model which says:

$$v_O = V_0 + g_m v_i$$

That is...our full output is based on our operating point/bias point ($V_0$) plus a small deviation in the input times a small signal gain $g_m$. We've done this for several other circuits before (diodes, MOSFETs in a common-source configuration)...let's do it for this configuration now.

gm is going to come from analyzing the first derivative of $v_O$ w.r.t. $v_I$ and analyzing it when it is $V_I$.

REMINDER THIS IS BASICALLY JUST THE FIRST TWO TERMS OF YOUR STANDARD TAYLOR/MACLAURIN SERIES EXPANSION

$$y = f(x) = f(x_0) + \frac{df}{dx}\bigg|_{x_0} (x-x_0) + \frac{1}{2!} \frac{d^2f}{dx^2}\bigg|_{x_0} (x-x_0)^2 + ...$$

or

$$y \approx f(x_0) + \frac{df}{dx}\bigg|_{x_0} (x-x_0) \quad \text{← just first two terms}$$
So this means from our original thing that since:

\[ V_o = V_I - \sqrt{\frac{2 \cdot \text{gain}}{k}} - V_T \]

\[ \left. \frac{dV_o}{dV_I} \right|_{V_I} = 1 \quad \text{.... antclimatic...} \quad \therefore = 1 \]

so \( g_m = 1 \quad \Rightarrow \quad V_o = 1 \cdot V_i \)

so small signal model

\[ \text{The small signal model is "very" simple..gain of 1} \]

\[ V_o \approx V_o \left|_{V_I} \right. + g_m \cdot V_i \]

next page \( \nu \)
We already found that large signal

\[ V_o = V_I - \sqrt{\frac{2I_{ss}}{k}} - V_T \]

we know that \( gm = 1 \) so overall this is saying:

\[ V_o \approx V_I - \sqrt{\frac{2I_{ss}}{k}} - V_T + v_i \]

And because \( v_I = V_I + v_i \) this is actually saying:

\[ V_o \approx V_I - \sqrt{\frac{2I_{ss}}{k}} - V_T \]

WHICH IS EXACTLY THE SAME TO OUR FULL MODEL WE ORIGINALLY DERIVED!!!

\[ V_o = V_I - \sqrt{\frac{2I_{ss}}{k}} - V_T \]

THIS IS WEIRD! THIS IS SAYING OUR FULL MODEL WITH A SMALL SIGNAL APPROXIMATION IS ACTUALLY EXACTLY THE SAME AS OUR FULL SIGNAL MODEL THAT WE ORIGINALY DERIVED AT THE BEGINNING!! THIS SHOULD SEE M ODD. THIS HAS NOT HAPPENED BEFORE WITH DIODES OR WITH OTHER MOSFET CIRCUITS!!!

The reason for this is that a small signal model is a way of linearly approximating a circuit’s operation around a known or inferred large signal operating point. If we go back and check out the plot we made earlier in these notes, you’ll see that in the region we care about the input to output relationship is already linear!!! It has a slope of 1 in fact in that plot, correct?

So when we tried to linearly approximate the relationship between \( v_I \) to \( v_O \), we accidentally captured all information...we weren’t approximating since a first order expressing is all that’s needed.

This is distinctly different than the case with diodes or even MOSFETs in source-follower topologies...in those cases, the input to output relationships ARE non-linear so the small signal approximation of the full signal:

\[ V_o \approx V_i + gm v_i \]

will actually be an approximation...in our case with the source follower, it turned out that the approximation was also accurate.
Aside...why do we even have these source follower amplifier circuits? I talked about it like it is "a thing" and it is. While at first glance a circuit that reproduces an input voltage at its output with a gain of 1 might seem useless, it allows us to buffer voltages! (just like with op amp buffers). This effectively gives us Power gain (as opposed to voltage gain), so we can provide a given voltage with a lot more current (job of the MOSFET) than maybe whatever circuit was providing $V_i$ could do originally.

In fact, and Prof Lang sort of went over this in lecture, but the ability and reason that op amp CAN be a buffer is because of this circuit...this circuit isn't actually "like" an op amp buffer...it is the buffer in the first place just like how Soylent Green is people.