In these notes, I present the method to be used when analyzing op-amp circuits. This is not intended to be a comprehensive introduction to op-amps, but rather a practical guide to their analysis, illustrating some useful examples.

The Basic Steps

There are four basic steps to analyzing an ideal op-amp:

1. Check that it is really ideal, i.e. that there is adequate negative feedback provided. Often in this course we will simply tell you that it is the case, or it will be obvious because only the negative terminal has feedback, but in more advanced classes you will learn how to do your own analysis in situations where it is less clear.

2. Assume that \( v_+ \approx v_- \) and that \( i_+ = i_- = 0 \).

3. Solve for the circuit parameters of interest based on these assumptions.

4. If you are not told in advance or unsure if op-amp is ideal, you should check at the end of your analysis that the output is indeed not saturated, i.e. that \( -V_S < v_{OUT} < V_S \). If that is not the case, the assumptions that went into your analysis were flawed, so the analysis is invalid.

Photodiode Readout

Here’s a worked example, showing how we apply these steps to a real-world circuit. A photodiodes (a sensor that detects light) is often modeled as a current source whose strength depends on the light intensity applied to it.\(^2\)

\[ I_{PD} \]

\[ R \]

\[ v_+ \]

\[ v_- \]

\[ v_0 \]

\[ \text{Photodiode Readout} \]

\(^1\) It is important to respect the approximation symbol here! Students can run into big problems when they assume perfect equality. For example, one cannot simply connect a wire between the ‘+’ and ‘–’ terminals, and expect your circuit to work. The tiny difference between the two inputs may be negligible for purposes of analysis, but it is critical to correct operation.

\(^2\) The equivalent circuit model for a photodiode is more complex than a simple current source, but often, and for our purposes, a current source will suffice.
1. Observing that there is only feedback to the negative terminal, it seems likely that the system can be treated as ideal.

2. Assuming that $v_+ \cong v_-$ we conclude that $v_- \cong 0$

3. We perform nodal analysis by summing the inputs at the negative input terminal.
   
   $$I_{PD} + (v_o - 0)G = 0$$
   
   $$\Rightarrow v_o = \frac{I_{PD}}{G} = -I_{PD}R$$

   A more conventional approach would be to use the “intuitive” method of analysis. By this method, one looks at the circuit and starts making simplifications one at a time until a solution presents itself.

   In this case, we observe that $I_{PD}$ must flow up towards and through the feedback resistor (because it cannot enter the - terminal. As a result, the voltage drop from $v_-$ to $v_o$ must be $-I_{PD}R$.

   We showed you the nodal analysis approach first, to help you see the connection between the two approaches. In future examples, we will only illustrate the intuitive method.

4. Because in this case we were not given actual numbers, if we hadn’t been informed in advance that the system was ideal, we would state that this treatment is valid only when

   $$|I_{PD}R| < V_S.$$  

An impossible device

Consider the following circuit:

![Circuit Diagram]

These devices are not realistic because

- You may have noticed that we did not do the node method at the output terminal. This was deliberate: the node method cannot be used at the output because the dependent source hidden inside each op-amp (remember the fundamental op-amp model) can source an arbitrary current, and thus is not sufficiently constrained to add useful information. Unfortunately, it is common for students to try this anyway, assume the output current of the opamp is zero (which it is generally not!), and end up in a mess.

3 Be careful to verify that there is negative feedback in the circuit before assuming $v_+ = v_-$. Circuits with positive feedback will often appear to provide the same analytic result as circuits with negative feedback, but due to the positive feedback we know the solution will be unstable—any slight fluctuation in $v_+ - v_-$ will immediately be amplified and the detector will end up at its rails. In this class, we will generally tell you in advance when it is safe to assume the opamp is ideal.
We are asked to find the Thévenin or Norton equivalent circuit when viewed at the terminal.\(^5\) We will use the test source method (assuming the op-amp is ideal as we will always do today). Here, we could also perform nodal analysis (at the + and - terminals) and solve for the unknowns, but that would be much slower than using our intuition.

Noticing first of all that if we put our ground reference at the Thévenin terminal, \(v_+ = v\), thus \(v_- = v\). Now let’s consider the current \(i\) entering the ‘+’ Thévenin terminal. Because \(i_+ = 0\) for op-amps, we know the current \(i\) must all go through \(R_1\). As a result, we can immediately conclude that at the output node of the op-amp, the potential is \(v - iR_1\). However, if we now look at the bottom branch, the two \(R\) resistors are just a voltage divider with \(v\) at the intermediate node. As a result, the output node of the op-amp must have potential \(2v\). Comparing these two relations, we find \(2v = v - iR_1\) so \(v = -iR_1\).

Because \(i\) and \(v\) are our port variables, we can conclude that \(R_{TH} = -R_1\). Thus we have demonstrated at negative resistor!!!\(^6\)

**How Is All This Possible**

It is very legitimate at this point to ask how is all this possible? After all, negative feedback seems to be a bit of a cheat—an answer to all our problems that is hard to quantify and understand.

Feedback is genuinely hard to wrap your head around. Basically, we’re used to thinking in a more stimulus, response style, where actions happen and we see consequences, as shown below.

\[ \text{IN} \quad \square \quad \text{OUT} \]

In this case, a signal travels from IN to OUT and is multiplied by a factor \(A\). There is no feedback in this system. Now let’s see what happens when we add feedback.

But with feedback, the consequence influences the action. As such, the behavior of the system can’t be reasoned out in a linear path.

The following sketch more accurately models an op-amp, including feedback to both the terminals.

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\(^5\) Note, one of our terminals is grounded. The current traveling through \(R\) to ground will not in general equal \(i\), because some current will be provided by the op-amp output.

\(^6\) While you likely learned in your Physics classes in high-school and college so far that a negative resistor cannot exist, that is true only for lumped-element resistors, e.g. components you can buy off the shelf. Because Thévenin and Norton’s theorems tell us that we can replace circuits at their terminals with simplified versions or equivalent models, it turns out that negative resistors can indeed exist, and can in fact be extremely useful. One common use is to cancel out loss in resonators, which we will deal with later on in the course. For now, we will just treat it as a curiosity.
Here, the feedback is summed with IN+ and IN- before the signal goes to the opamp inputs. This circuit will only be stable (i.e. will exhibit negative feedback) if $\alpha_- > \alpha_+$, i.e. if the feedback to IN- dominates over the feedback to IN+. Imagine you are sitting at IN- and the signal goes up slightly (say by $\Delta$), the output will start to drop, say in the initial phase of the feedback, it drops by $\delta$ initially. Before the output can drop more, feedback the feedback kicks in and lowers the negative input to the amplifier $A$ by $\alpha_- A \delta$ (and raises the positive input by $\alpha_+ A \delta$). But now those changing inputs will change the output! So we have to repeat this analysis. If we assume $\alpha_+ = 0$, soon $A \alpha_- \delta = \Delta$, the two inputs to the amplifier will be equal, the output will be 0, and the feedback signal will stop. As a result, the negative feedback acts to shut down any changes in the inputs, and stabilizes the system.\footnote{Of course, as mentioned above, this is a simplified picture, and not intended to fully satisfy your curiosity. You can take a class like 6.302 Feedback Systems, to learn it properly. Suffice it to say that feedback and control are some of the most important concepts used in technology in the modern era.}

Once a system is stable, it is quite reasonable to conclude that $v_+ \cong v_-$. One way to see this is to consider the input/output characteristics of an isolated op-amp. This characteristic is known as the “transfer function”, and you may remember we saw it in class in a demo.

Because $V_S$ is typically about 10 V and $A$ is typically at least $10^5$, for the output to be anything other than saturated at the positive or negative rail, $v_+ - v_-$ must be less than or on the order of $V_S/10^5 \sim 100 \mu V$, i.e. $v_+ \approx v_-$.\footnote{Of course, as mentioned above, this is a simplified picture, and not intended to fully satisfy your curiosity. You can take a class like 6.302 Feedback Systems, to learn it properly. Suffice it to say that feedback and control are some of the most important concepts used in technology in the modern era.}

### Real-World Linear Dependent Sources

One of the fun and useful things to do with op-amps is to make real-world dependent sources of various types. We’ll show you two
Here, you have to imagine some load put across the feedback port, and ask yourself, what is the Thévenin equivalent circuit of the op-amp network viewed from that port? Here the $R_L$ is not part of our network, it is part of the external circuit.

Given some $i$ through that port and $v$ across it, we are looking to discover a relationship between them. As always, we could use the node method⁸, but there is an easier way. First, observe that $v_- \approx 0$. That tells us that the current through $R$ (left to right) will be $-V_i/R$. Because no current can enter the - port of the op-amp, this current must travel through the feedback resistor. Thus $i = V_i R$. In other words, this is a programmable current source!

Another dependent source can be demonstrated by making a few changes to the circuit above. Again, we are asking what the op-amp network looks like (i.e. the Thévenin or Norton equivalent) viewed from the labeled port.⁹

> Remember, not at the output port!

⁸ We label these ports with the current entering the positive terminal of the op-amp network because we are thinking of the entire network as acting as a component, thus correct labeling by the associated variables convention mandates the current entering the positive terminal. If we were asking for the equivalent of the load, we would turn the current arrow around.

⁹ We label these ports with the current entering the positive terminal of the op-amp network because we are thinking of the entire network as acting as a component, thus correct labeling by the associated variables convention mandates the current entering the positive terminal. If we were asking for the equivalent of the load, we would turn the current arrow around.
(again because current cannot enter the op-amp inputs), we find the output voltage equals \( v - iR_2 \). Equating these two expressions, we find \( i = -I_o R_1 / R_2 \). That is, the op-amp network acts as a current source sinking \(-I_o R_1 / R_2\), or sourcing \(I_o R_1 / R_2\). In effect, we’ve made a current amplifier!

Because a simple non-inverting amplifier can be thought of as a basic voltage controlled voltage source (VCVS), and the photodiode is a Current Controlled Voltage Source (CCVS), with an op-amp and a few resistors, we can construct all the dependent sources you’ve seen so far!

**Conclusions**

Opamps are primarily useful because they provide powerful functional capability in a circuit while remaining fairly simple to analyze.

Until now, we had to just postulate that things like current sources and dependent sources were possible. Now, we know how to make these circuit elements using opamps.

The analytic simplicity comes about as a result of negative feedback. There are useful op-amp circuits that do not use negative feedback, and we may look at some of those later in the class, but when negative feedback applies, the assumption of ideality provides massive simplification.

**Glossary and Definitions**

**Amplifier**  A circuit element that increases any signal parameter (generally voltage in this class). Generally, power must be supplied from an external source.

**Amplifier Transfer Function**  Output signal vs. input signal for an amplifier. For a simple linear amplifier this is characterized by a simple line through the origin whose slope is called the "gain". Non-linear amplifiers are more complicated, and the transfer function is sometimes plotted.

**Gain**  The amplification factor by which a signal is multiplied in an amplifier.

**Ideal Op Amp**  A concept used in circuit analysis of an op-amp without saturation rails, with exactly zero current into the input terminals, and with infinite gain.

**Lumped Element**  A simple component (i.e. single element) in which the electric and magnetic fields are assumed to be entirely local to the device.
Photodiode A light sensor consisting of a piece of silicon engineered to generate current when illuminated. Useful in a range of applications (e.g., the sensors that protect your hands from getting caught in an elevator door!).

Saturation A level (either positive or negative) which an amplifier cannot exceed even though inputs are increased. Saturation in an op-amp is often near the power supply levels (positive and negative).

Rail Saturation levels for an op-amp or other electronic system. Often used to refer to the power supply levels.