Name: ________________________

Solutions

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Time: 10 11 12 1

- There are 16 pages in this quiz, including this cover page.
- Please put your name in the space provided above, and circle the name of your recitation instructor together with the time of your recitation.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. When finished, clearly indicate your answer, perhaps by circling it.
- This is a closed-book quiz, but calculators and a single two-sided page of notes are allowed.
- Good luck!

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Problem 1: Linear Resistor/Source Circuits – 25%

The problem studies circuits involving independent sources, linear resistors and linear dependent sources. Note that the circuit in Part (A) is used again in Parts (B) and (C), though it is extended in the process.

(1A) Determine and draw the Thevenin equivalent for the circuit shown below as viewed from the indicated terminals. *Numerical results with appropriate units are expected.*

\[ e_1 = \frac{15}{45}(-15) = -5\, \text{V} \]
\[ e_2 = \frac{30}{45}(+15) = 10\, \text{V} \]
\[ \therefore V_{oc} = e_1 + e_2 = 5\, \text{V} \]

\[ R_{TH} = (15\, ||\, 30) + (15\, ||\, 30) = 20\, \Omega \]
A 20-Ω resistor is now connected to the circuit from Part (A) as shown below. What is the Thevenin equivalent of the modified circuit as viewed from the indicated terminals? 

Numerical results with appropriate are expected.

\[ V_{oc} = \frac{20}{40} (5V) = 2.5V \]

\[ R_{TH} = 20 || 20 = 10 \Omega \]

\[ 2.5V \]
The circuit from Part (A) is again modified, this time by the addition of a dependent source as shown below. What is the Thevenin equivalent of the modified circuit as viewed from the indicated terminals? Numerical results with appropriate units are expected.

\[ V_{oc} = 5V \]
\[ i' + 4i = 0 \]
\[ \therefore i' = 0 \]
\[ V_{oc} = 5V \]
\[ I_{sc} = -\frac{5}{20} = -0.25A \]
\[ 4i = -1A \]
\[ I_{sc} = -i' - 4i = -\frac{5}{4}A \]
\[ R_{TH} = \frac{5V}{\frac{5}{4}A} = 4\Omega \]
Problem 2: First-Order Circuits – 30%

This problem studies the first-order circuit shown below.

\[
\frac{dv_C(t)}{dt} + \frac{v_C(t)}{\tau} = \alpha i_S(t)
\]

Determine \( \alpha \) and \( \tau \). Express your answers in terms of \( R_1, R_2 \) and \( C \).

Node method \( \Rightarrow \)

\[
C \frac{dN_C}{dt} + \frac{V_C - R_1 i_S}{R_1 + R_2} = 0
\]

\[
\Rightarrow \frac{dN_C}{dt} + \frac{V_C}{C(R_1 + R_2)} = \frac{R_1}{C(R_1 + R_2)} i_S
\]

\[
\Rightarrow \tau = (R_1 + R_2) C \quad \& \quad \alpha = \frac{R_1}{(R_1 + R_2) C}
\]
(2B) Assume that the current source produces the constant current $i_s = I_A$, and that it has done so for a very long time. Determine the corresponding steady-state value of the capacitor voltage $v_C$. Express your answer in terms of $\alpha$, $\tau$ and $I_A$.

$$\text{Steady state } \Rightarrow \frac{d}{dt} \rightarrow 0$$

$$\Rightarrow v_C = \alpha \tau i_s = \alpha \tau I_A$$
Following the steady-state condition of Part 2B, the current source steps from a value of \( i_S = I_A \) to a value of \( i_S = I_B \) at \( t = 0 \). Determine the capacitor voltage \( v_C(t) \) for \( t \geq 0 \). Express your answer in terms of \( \alpha \), \( \tau \), \( I_A \) and \( I_B \).

\[
\begin{align*}
    v_C(0) &= \alpha \tau I_A \\
    v_C(\infty) &= \alpha \xi I_B \\
    \text{Time constant} &= \tau \\
    v_C(t) &= \alpha \tau I_A e^{-t/\tau} + \alpha \tau I_B \left(1 - e^{-t/\tau}\right) \\
    &= \alpha \tau I_B + \alpha \tau (I_A - I_B) e^{-t/\tau}
\end{align*}
\]
(2D) Assume that at $t = 0^-$, the capacitor voltage is given by $v_C(0^-) = V_o$. Further assume that the current source produces the current impulse $i_s(t) = Q_o \delta(t)$ at $t = 0$. Determine the capacitor voltage $v_C(t)$ for $t > 0$. Express your answer in terms of $\alpha$, $\tau$, $V_o$ and $Q_o$.

\[
\begin{align*}
0^+ & \quad \int_{0^-}^0 \left[ \frac{d}{dt} \frac{\Delta Q}{C} + \frac{V_C}{C} \right] dt = \alpha Q_o \delta(t) \\
\downarrow & \\
V_C(0^+) - V_C(0^-) + V_o & = \alpha Q_o \\
\uparrow & \\
V_o & \\
\Rightarrow & \quad V_C(0^+) = V_o + \alpha Q_o \\
\Rightarrow & \quad V_C(t) = \left[ V_o + \alpha Q_o \right] e^{-t/\tau} \quad t \geq 0
\end{align*}
\]