Lecture 26

Today

1. Wireless power transfer

\[ 0_m = j\omega M(i_1 + i_2) \]

\[ 0_1 = j\omega M(i_1 + i_2) + j\omega (L-M)i_1 \]

\[ = j\omega l_1 + j\omega m_2 \]

Take limits

\[ k \to 0 \implies M \to 0 \]

\[ k \to 1 \implies M = L \]

\[ \frac{\omega_M}{R_S} = \frac{\omega_M L}{R_S} = k \left( \frac{\omega_M L}{R_S} \right) = kQ \]

\[ M = kV_1L \]

\[ M = kL \]

Matched resonators

\[ \frac{\omega_M}{R_S} = \frac{\omega_M^2}{R_S^2} = \frac{2\omega}{R_S} \]

Use high Q to compensate for small k

\[ \eta = \frac{\text{Power at load}}{\text{Power from source}} = \frac{1}{2} \frac{\omega_M^2 M^2}{R_S^2} = \frac{1}{2} k^2 Q^2 \]
Wireless power transfer

- Cut the cord on power, not just data
- Most are electrical
- Many use inductive coupling

Coupled inductors

Some flux $\phi$ from one coil couples to other coil

$M$: mutual inductance
$M$ represents coupling $\sim \phi_{21}$

$\begin{align*}
[v_1] &= \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} [i_1] \\
[v_2] &= M \frac{di_1}{dt}
\end{align*}$

$M = k\sqrt{L_1 L_2}$

$k$ is coupling coefficient $[0..1]$
Ideal transformer

If \( k \to 1 \)
\[
M = \sqrt{L_1 L_2}
\]
Perfect coupling aka all flux from coil 1 goes to coil 2 and vice versa

From before:
\[
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix} =
\begin{bmatrix}
\jmath \omega L_1 & \jmath \omega M \\
\jmath \omega M & \jmath \omega L_2
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
\]
\[
v_1 = \jmath \omega L_1 i_1 + \jmath \omega M i_2 \Rightarrow i_1 = \frac{v_1 - \jmath \omega M i_2}{\jmath \omega L_1}
\]
\[
v_2 = \jmath \omega M i_1 + \jmath \omega L_2 i_2 
\]
\[
v_2 = \jmath \omega L_2 i_2 + \frac{\jmath \omega L_2}{\jmath \omega L_1} v_1 - \frac{\jmath \omega M j \omega M i_2}{\jmath \omega L_1}
\]
\[
v_2 = \jmath \omega L_2 i_2 + \frac{L_2}{L_1} v_1 - \frac{\jmath \omega L_2 i_2}{L_1}
\]

Wireless power transfer

• When you can be close and control orientation, tight coupling \((k \sim 1)\) is good
Wireless power transfer

• When you can be close and control orientation, tight coupling ($k \sim 1$) is good

https://9to5mac.com/2019/02/13/multi-coil-charger/
https://pt.ifixit.com/Teardown/iPhone+XR+Teardown/114123

Coupled inductors

Assume:
$L_1 = L_2 = L$

This circuit obeys the same relation as the one on the left...but easier to analyze!
Wireless power transfer challenge

- Transfer power from source to load
- In this case $k \ll 1$

![Image of wireless power transfer device]

![Plot of inductance ratio $\frac{L_2}{L_1} = k$ vs. center-center separation in cm]

Wireless power transfer challenge

- How to maximize energy transfer from source to load?

$k \ll 1 \Rightarrow M \ll 1$

Coupled coils

- Voltage Divider
- Voltage Divider

Load
Use coupled resonators

- Add $C_S$ on source side

Choose $C_S$ and $\omega$ so that: $\frac{1}{\omega_0 C_S} = \omega L$

Thevenin equivalent: $V_{TH} = \frac{j\omega M}{j\omega L + \frac{1}{j\omega C_S} + R_S} V_S = \frac{j\omega_0 M}{R_S} V_S$ at resonance

$$Z_{TH} = j\omega (L - M) + \frac{(j\omega (L - M) + \frac{1}{j\omega C_S} + R_S)j\omega M}{j\omega L + \frac{1}{j\omega C_S} + R_S}$$

$$Z_{TH} = j\omega_0 (L - M) + \frac{(-j\omega M + j\omega L + \frac{1}{j\omega C_S} + R_S)j\omega M}{j\omega L + \frac{1}{j\omega C_S} + R_S}$$

$$Z_{TH} = j\omega_0 (L - M) + \frac{(-j\omega_0 M + R_S)j\omega_0 M}{R_S}$$ at resonance

$$Z_{TH} = j\omega_0 L - j\omega_0 M + \frac{\omega_0^2 M^2 + j\omega_0 M R_S}{R_S}$$

$$Z_{TH} = j\omega_0 L + \frac{\omega_0^2 M^2}{R_S}$$
Use coupled resonators

• Add $C_L$ on load side

\[
\frac{\omega_0^2 M^2}{R_S} j \omega_0 M \frac{R_S}{V_S} + L + j \frac{R_L}{V_L}
\]

Choose $C_L$ and $\omega_0$ so that: $\frac{1}{\omega_0 C_L} = \omega_0 L$

Then, for maximum power transfer, choose: $R_L = \frac{\omega_0^2 M^2}{R_S}$

*Double resonators!*

---

Use coupled resonators

• The overall circuit

Use coupled resonators to increase voltage from $V_S$ so that even if $M$ is small, substantial voltage & power can get to $V_L$
Drive some LEDs

- Use parallel $C_L$ to maximize voltage rather than power

\[ L = 230 \, \mu H \quad C_s = C_L = 9.6 \, nF \quad \omega_0 \sim 107 \, kHz \]
\[ R_s = 50 \, \Omega \quad R_L \sim 1 \, k\Omega \ldots 5 \, k\Omega \]
\[ M \sim \frac{L}{10} \ldots \frac{L}{200} \]

\[ V_S = 10 \, V_{pk} \]

Primary $Q = 3 \quad \Rightarrow V_P = 30 \, V_{pk}$

Secondary $Q = \frac{R_L}{Z_0} \approx 20$

From holiday lights lab

Tesla coil

- Air core for low core loss, high breakdown voltage & high resonance frequency
- Short primary and long secondary for high turns ratio
- Loose coupling between primary and secondary

http://wiring.mx.tl/6-volt-wiring-diagram-for-tesla.html
Tesla coil

- Air core for low core loss, high breakdown voltage & high resonance frequency
- Short primary and long secondary for high turns ratio
- Loose coupling between primary and secondary

http://wiring.mx.tl/6-volt-coil-wiring-diagram-for-tesla.html