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12/3/19

Lecture 24

Nomenclature

TODAY

1. MOSFET amp
2. S.s. Circuit
3. Two-stage amp

![Diagram of nomenclature with waveforms and voltage levels]

1. MOSFET amp

in saturation:

$$i_D = \frac{K}{2} (V_{GS} - V_T)^2$$

$$V_{out} = V_{DD} - i_D R_{pu}$$

$$V_{out} = V_{DD} - \frac{K R_{pu}}{2} (V_{IN} - V_T)^2$$

2. Small-signal circuit

- Linear resistor

slope at a.p. = \frac{1}{R}

3. Linearize DC voltage source

- MOSFET amp

![Diagram of MOSFET amplifier with resistor and voltage levels]

slope at a.p. = \infty \Rightarrow S_a = e_0

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\[ V_{\text{out}} = -g_{m} R_{pu} V_{\text{in}} \text{ same as before} \]

\[ 11-13 \]

Two-stage amplifier

- Large Signal
  
  \[ V_{\text{mid}} = V_{\text{DD}} - \frac{K_{1}R_{1}}{2} (V_{\text{in}} - V_{T})^{2} \]
  
  \[ V_{\text{out}} = V_{\text{DD}} - \frac{K_{2}R_{2}}{2} (V_{\text{mid}} - V_{T})^{2} \]
  
  \[ V_{\text{out}} = V_{\text{DD}} - \frac{K_{1}R_{1}}{2} \left[ V_{\text{DD}} - \frac{K_{1}R_{1}}{2} (V_{\text{in}} - V_{T})^{2} - V_{T} \right]^{2} \]

- No interstage loading b/c \( I_{Q} = 0 \)

\[ V_{\text{mid}} = -g_{m1} R_{1} V_{\text{in}} \]

\[ V_{\text{out}} = -g_{m2} R_{2} V_{\text{mid}} \]

\[ V_{\text{out}} = g_{m} g_{m2} R_{1} R_{2} V_{\text{in}} \]
MOSFET Amplifier

MOSFET as small signal amplifier

• Changing the operating point (the “bias” point) $V_{GS}$ results in different gains

Add an offset to input signal

Linear Term of Taylor Series Expansion

Operating Point “Bias” Point

Add an offset to input signal
Taylor Series...Review

\[ y = f(X) + \frac{df(x)}{dx} \bigg|_x \cdot (x(t) - X) + \frac{1}{2} \frac{d^2f(x)}{dx^2} \bigg|_x \cdot (x(t) - X)^2 + \ldots \]

If deviations are small, this second-order term (and higher ones) can be ignored

The deviation \( x(t) - X \) around our point of interest is the “small-signal”

The point around which we do our Taylor Series \( X \) the “bias point” or “operating point”

Applying Taylor Series to MOSFET amp

- Input-output relationship of MOSFET amplifier in saturation:
  \[ v_{OUT} = f(v_{IN}) = V_{DD} - \frac{KR_{PU}}{2} (v_{IN} - V_T)^2 \]

- Take the Taylor Series Expansion around the operating point \( V_{GS} \) we get:
  \[ v_{OUT} = f(V_{IN}) + \frac{df(v_{IN})}{dv_{IN}} \bigg|_{V_{IN}} \cdot (v_{IN}(t) - V_{IN}) + \ldots \]
  Operating point “DC”
  \[ v_{IN}(t) = V_{IN} + v_{in}(t) \]
  Excursion “s.s”

\[ v_{OUT}(t) = V_{OUT} - KR_{PU} (V_{IN} - V_T) \cdot (v_{IN}(t) - V_{IN}) \]

\[ v_{IN}(t) = V_{IN} + v_{in}(t) \]
\[ v_{OUT}(t) = V_{OUT} + v_{out}(t) \]

\[ v_{out}(t) = -KR_{PU} (V_{IN} - V_T) \cdot v_{in}(t) \]

Constant

Small-signal gain

* We’ll cover this nomenclature on next slide
Applying Taylor Series to MOSFET

**Linear Term of Taylor Series Expansion**

\[ v_{OUT} \approx f(V_{IN}) + \frac{df(V_{GS})}{dv_{GS}} \bigg|_{V_{IN}} \cdot v_{in}(t) \]

**Total**

\[ v_{OUT}(t) = V_{OUT} + v_{out}(t) \]

**Every branch voltage and current will now have a Small Signal and Bias Point**

- Bias point solved using load-line analysis, etc.
- Small signal comes from linearizing around bias point
Applying Taylor Series to MOSFET

- I-V relationship of MOSFET device in saturation:
  \[ i_D = f(v_{GS}) = \frac{K}{2} (v_{GS} - V_T)^2 \]
- Take the Taylor Series Expansion around the operating point \( V_{GS} \) we get:
  \[ i_D = I_D + K(V_{GS} - V_T) \cdot (v_{GS}(t) - V_{GS}) \]

\[ i_d(t) = K(V_{GS} - V_T) \cdot v_{gs}(t) \]
\[ i_d(t) = g_m \cdot v_{gs}(t) \]

**LINEAR!**

Operating point  
**“DC”**
\[ v_{GS}(t) = V_{GS} + v_{gs}(t) \]

Excursion  
**“s.s”**
\[ i_D(t) = I_D + i_d(t) \]

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Linearizing voltage source

- I-V relationship of voltage source:
  \[ v_V = f(i_V) = V_{DD} \]
- Take the Taylor Series Expansion around the operating point \( V_{GS} \) we get:
  \[ v_V = f(i_V) + \frac{df(i_V)}{di_V} \bigg|_{i_V} \cdot (v_V(t) - V_V) + \cdots \]

\[ v_V = V_{DD} + 0 \cdot (v_V(t) - V_V) = V_{DD} + 0 \cdot 0 \]

\[ v_{V}(t) = V_{DD} + v_{V}(t) \]

\[ v_{V}(t) = 0 \]  
short circuit

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Small signal circuit equivalents

1. Carry out a (nonlinear) large-signal operating-point analysis (analytical, graphical, numerical).
   - The small-signal model operates around this bias, showing variations from this bias.
2. Replace each element by its linearized version
3. Solve small-signal circuit
   - Use any linear technique: Thevenin, impedance, etc.
4. Get total response if desired
Total response

• The full response is the bias voltage PLUS the small signal voltage

\[ v_{OUT} = V_{DD} - I_D R_{PU} - g_m R_{PU} v_{in} \]

What’s missing

• How to choose bias point? Maybe to get a certain dynamic range before transistor leaves saturation
• Get particular gain
• Set output operating-point voltage
Some numbers

\[
v_{\text{out}} = -g_m R_{PU} v_{\text{in}} = -K (V_{IN} - V_T) R_{PU} v_{\text{in}}
\]

MOSFET in saturation: \( I_D = \frac{K}{2} (V_{IN} - V_T)^2 \Rightarrow V_{IN} - V_T = \sqrt{\frac{2I_D}{K}} \)

s.s. gain: \( -K R_{PU} \sqrt{\frac{2I_D}{K}} = -R_{PU} \sqrt{2KI_D} \)

Set bias point so that: \( V_{OUT} = \frac{V_{DD}}{2} \Rightarrow R_{PU} I_D = \frac{V_{DD}}{2} \Rightarrow I_D = \frac{V_{DD}}{2R_{PU}} \)

s.s. gain: \( -R_{PU} \sqrt{2KI_D} = -\sqrt{R_{PU} K V_{DD}} \)

\( V_{DD} = 5 \text{ V} \)
\( K = 0.14 \text{ A/V}^2 \)
\( R_{PU} = 1 \text{ kΩ} \)

s.s. gain: \( \approx -26 \)