Switching Supplies II
Boost Converters and Diodes!

DC-DC boost converters

- Sometimes you want to create a large DC voltage from a small DC voltage

Camera flash (Xenon bulb)
Bug zapper
Stun gun

~3V to 100's of Volts!

Motivation

- Transferring power over grid is best done at high voltages.
  - Need a way to efficiently down-convert for local electronics (which work at low voltages)
  - Buck converters allow us to do that!
- Mobile devices rely on batteries to store energy
- Batteries often produce their energy using lower voltages (few volts usually, Lipo Battery is at 3.7V nominally)
- Some electronics need voltages higher than 3.7V:
  - Backlight, EEPROM (Flash), many sensors (gyroscopes)
- Need a way to convert voltages up, and ideally do so efficiently
- Boost converters are one way to do this

~3.7V to 20V!
Buck Converter (from Tuesday)

• Takes an input voltage and produces a **lower** output voltage

![Buck Converter Diagram]

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Boost Converter

• Takes an input voltage and produces a **higher** output voltage

![Boost Converter Diagram]

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Boost Converter

• The diode and the FET act like switches just like before in the Buck Converter!

![Boost Converter Diagram]

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Drive with a Periodic Signal

• Variable Duty Cycle!! (we’ve seen this before)

![Drive with a Periodic Signal Diagram]

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• In the Buck converter, your output voltage would be (approx): \( \frac{v_{IN}}{D} \)

• What about Buck?
Stage 1
• $S_1$ closed, $S_2$ open.

Stage 2
• $S_1$ open, $S_2$ closed.

Stage 3
• $S_1$ open, $S_2$ open.

• $S_1$ is closed, $S_2$ is open!
• Current builds up through the inductor: $i_L(t) = \frac{1}{L} \int v_{IN} \, dt$
  • Since $v_{IN}$ is constant: $i_L(t) = v_{IN} \cdot t$
• Therefore energy is being built up and stored in the inductor!
During Stage 1

- The Capacitor and Load Resistor are Isolated, forming their own cozy RC circuit away from the whole world...voltage on cap just decay

\[ v_C(t) = v_C(0)e^{-t/RC} \]

- The voltage source and inductor are isolated, building up energy

\[ i_L(t) = v_{IN}t \]

Stage 2 Math: Where does the current go?

\[ i_L(t) \rightarrow i_C(t) + i_L(t) \] Either is fine. R consumes power, C stores it for consumption later by R. So how much \( i_L \) moves over during Stage 2???

\[ i_L(t) = i_L(DT) + \frac{1}{L} \int_{DT}^{T} (v_{IN} - v_C(t)) \, dt \]

Current built up at end of stage 1

- If \( v_C \) is small/lower than \( v_{IN} \) during Stage 2, \( i_L \) will stay positive and grow!

- Much of that current dumps into the capacitor, however, and because \( v_C(t) = \frac{1}{C} \int i_C \, dt \) that cap voltage will build up!

- If \( v_C \) is larger than \( v_{IN} \) during Stage 2, \( i_L \) will potentially drop to 0 even if it leaves Stage 1 positive!

Stage 2

- \( S_1 \) opens and \( S_2 \) closes at some point in time \( t=DT \)
- Right before switch\(^1\): \( i_L(DT^-) = \frac{1}{L} v_{IN} \cdot DT \)
- Inductors don’t want much in life, but they do want \( i_L(t) \) to be continuous

\[ i_L(t) \]

\[ v_{IN} \]

\[ v_{OUT} \]

\[^1\] Assuming current started at 0...otherwise this is the delta current built up during period

Stage 3

- As long as \( i_L(t) \) is positive, \( S_2 \) stays closed letting current (energy) move to the right half of the circuit, but when \( i_L(t) \) gets low enough, \( S_2 \) will open, locking that sweet energy in.
- Whatever voltage/energy is remaining on the capacitor can now only be dissipated by the load resistance (nothing can flow back to the inductor)!! Yahtzee!

\[ v_{IN} \]

\[ v_{OUT} \]
During Stage 2

- The inductor dumps its energy into the cap:
  - Inductor current drops
  - Capacitor voltage builds
- How do they change?
- Depends on the circuit coefficients $\alpha$ and $\omega_0$

If $\alpha < \omega_0$ for example...

- The inductor dumps its energy into the cap:
  - Inductor current drops
  - Capacitor voltage builds
- How do they change?
- Depends on the circuit coefficients $\alpha$ and $\omega_0$

Math/treatment very similar
To "LC and some switches, Take 1" from EX06

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### Stage 2 Math: How does the current go?

$$i_L(t) = i_C(t) + i_R(t)$$

$$i_L(0^+) = \frac{1}{L} \int_{0^+}^{t} (v_{IN} - v_C(t)) \, dt = \frac{C}{R_L} \frac{dv_C(t)}{dt} + \frac{v_C(t)}{R_L}$$

$$\frac{v_{IN}}{L} - \frac{v_C(t)}{L} = C \frac{d^2 v_C(t)}{dt^2} + \frac{1}{R_L} \frac{dv_C(t)}{dt}$$

$$\frac{v_{IN}}{L} = \frac{d^2 v_C(t)}{dt^2} + \frac{1}{R_L} \frac{dv_C(t)}{dt} + \frac{1}{LC} v_C(t)$$

Response will therefore be one of three forms depending on what $\alpha$ and $\omega_0$ are!

<table>
<thead>
<tr>
<th>$\alpha &lt; \omega_0$</th>
<th>$\alpha = \omega_0$</th>
<th>$\alpha &gt; \omega_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_C(t) = A_0 e^{-\alpha t} \cos (\omega_d t - A_1)$</td>
<td>$v_C(t) = A_0 e^{-\alpha t} + A_1 e^{-s_1 t}$</td>
<td>$v_C(t) = A_0 e^{-\alpha t} + A_1 e^{-s_2 t}$</td>
</tr>
</tbody>
</table>

Initial Conditions! Yield these terms!

If circuit is extremely underdamped
(assume no load resistance)
($\alpha << \omega_0$)

$$v_C(t) = A_0 e^{-\alpha t} \cos (\omega_d t - A_1)$$

$\alpha$ tiny so $e^{-\alpha t}$ disappears and $\omega_d \approx \omega_0$

$$v_C(t) = A_0 \cos (\omega_d t - A_1)$$

Cos + phase form

$$v_C(t) = \frac{v_{IN}}{L} + \sqrt{\frac{v_C(0) - \frac{v_{IN}}{L}}{\frac{L}{C} i_L(0)^2}} \cos \left( \omega_d t - \tan^{-1} \left( \frac{\frac{L}{C} i_L(0)}{v_C(0) - \frac{v_{IN}}{L}} \right) \right)$$

$$i_L(t) = -\frac{C}{L} \sqrt{\frac{v_C(0) - \frac{v_{IN}}{L}}{\frac{L}{C} i_L(0)^2}} \sin \left( \omega_d t - \tan^{-1} \left( \frac{\frac{L}{C} i_L(0)}{v_C(0) - \frac{v_{IN}}{L}} \right) \right)$$

Initial Conditions! Yield these terms!
Cyclic Operation

Quasi-qualitative plots of operation:

Cap starts with no voltage across it and that voltage gradually builds up!

Eventually decay from load will eventually cancel gains from inductor current injection! And that’s ok

The system will reach a steady-state operation

Just like in the PWM circuits!

What will that steady-state voltage be?

http://ctms.engin.umich.edu/CTMS/index.php?aux=Activities_BoostcircuitA
Let's assume that our Boost converter reaches some sort of steady-state, meaning that at the beginning and end of a period (T), the voltage across the capacitor and the current through the inductor are the same.

During Stage 1 (when the inductor is "charging" up), it is in a circuit that looks like this:

\[ \dot{i}_L(t) = \dot{i}_L(0) + \frac{V_{\text{in}}}{L} \int_0^{\frac{dT}{2}} (V_{\text{in}} - V_C(t)) \, dt \]

\( V_{\text{in}} \) is a constant, so at the end of the Stage 1 (at \( t = DT \)):

\[ \dot{i}_L(0) = \dot{i}_L(0) + \frac{V_{\text{in}} \cdot DT}{L} \]

\( \Delta i_L \)

In Stage 2, the time-domain expression for \( i_L \) is now based on what the capacitor voltage is!

\[ \dot{i}_L(t) = \dot{i}_L(0) + \frac{V_{\text{in}} \cdot DT}{L} + \int_{\frac{DT}{2}}^{T} (V_{\text{in}} - V_C(t)) \, dt \]

If we assume that the capacitor is large enough to not vary much we can treat \( V_C(t) \) as a constant, making the integration easy.

\[ \dot{i}_L(T) = \dot{i}_L(0) + \frac{V_{\text{in}} \cdot DT}{L} + \frac{1}{C} \left( V_{\text{in}} - \left< V_C \right> \right) (T - DT) \]

\[ \left< V_C \right> = \left< -V_C \right> \]

If we want the finishing current to be the same as the starting current (assuming the system is in a stable, cyclic behavior), then \( i_L(T) = i_L(0) \)!

\[ \dot{i}_L(T) = \dot{i}_L(0) = \dot{i}_L(0) + \frac{V_{\text{in}} \cdot DT}{L} + \frac{V_{\text{in}} - V_C}{L} (T - DT) \]

\[ \downarrow \]

\textit{NEXT PAGE...}
Left with something pretty simple:

\[ 0 = \frac{v_{in}T}{T} - \langle v_c \rangle (1-DT) \]

\[ T(1-D) \langle v_c \rangle = v_{in}F \]

\[ \langle v_c \rangle = \frac{v_{in}}{(1-D)} \]

\( D \) varies from 0 to 1...so this means our capacitor's average voltage (which is our output!) will vary from \( v_{in} \) to infinity!!!

In reality other things, non-idealities get in the way, but under certain circumstances, this expression holds true...by varying our duty cycle we can get higher output voltages than what we put in!!!
Cyclic Operation

- Depending on $D$, $T$, $\alpha$ and $\omega_0$ ($R_L, L, C$) you can get different behaviors!
- $T - DT$ shorter? Current might not drop all the way to 0?
- Current will eventually reach cycle-to-cycle steady state average value

Cyclic Operation

- Depending on $D$, $T$, $\alpha$ and $\omega_0$ ($R_L, L, C$) you can get different behaviors!
- More damping? (power-hungry load, aka $R_L$ is low)...
- Current will drop quick and may hit 0

Cyclic Operation

- S2 opens to prevent current going negative

Cyclic Operation

- Depending on $D$, $T$, $\alpha$ and $\omega_0$ ($R_L, L, C$) you can get different behaviors!
- Waaay too much damping? (power-hungry load, aka $R_L$ is low)...

11/14/19
Lots and Lots of Parameters to Optimize

• 6.131 and 6.334 spend their time on this stuff!

Booost converter

Under the Hood of a DC/DC Boost Converter
by Brian T. Lynch included as supplemental fun reading

Costs a few dollars!

MIT Ion plane

• aka flying boost converter
• The key
  • 200 V → 40 kV
  • Low mass: 1.2 kW/kg

What about those “magic” parts?

The so-called “diode” and “transistor” that were doing all this switching? How are they working?
Transistor

• Go into them a lot more in coming weeks
• Three terminal device:
  • Middle terminal enables/disables connectivity between two outside ones

Diode

• Historically it was the first **Non-Linear** device we as humans developed and worked with (whether it was known at the time or not)
• What does it meant to be non-linear?

\[ i = I_S \left( e^{\frac{v}{kT}} - 1 \right) \]

Diode Can Act Like a Switch

• Voltage in one way: Conduct (act like a wire)
• Voltage in the other way: Don’t conduct (act like an open)

Diode Can Act Like a Switch

• Current Positive? Let it through
• Current Negative? Block it!

[https://www.electronics-tutorials.ws/diode/diode_5.html](https://www.electronics-tutorials.ws/diode/diode_5.html)
Diodes are One Way Electrical Valves

- Use them all the time in Power Conversion, Signal Processing, Logic, Computation!

Why are Diodes so Great?

- So far in class what can we do with voltages or currents (with Rs, Ls, and Cs?)
  - We can:
    - Scale them (transfer functions)
    - Add them
    - Subtract them
    - Integrate them
    - Differentiate them
  - We can’t:
    - Multiply them
    - Divide them

A Non-Linear Electrical Device

- Why is a non-linear electrical device important?
- Think about a Taylor Series
- Model some function \( f(x) \) around some point \( c \)
- That function can be approximated/expressed as:

\[
\sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!} (x - c)^k
\]

\[
= f(c) + f'(c)(x - c) + \frac{f''(c)}{2}(x - c)^2 + \cdots
\]

0th order term  1st order term  2nd order term

A Non-Linear Electrical Device

- If \( x \) is actually sum of two values (maybe two voltages or two currents) and we expand out that second order term you’ll get:

\[
\frac{f''(c)}{2}(v_a + v_b - c)^2 + \frac{f''(c)}{2}(v_a^2 + 2v_a v_b - 2v_a c - 2v_b c + c^2) + \cdots
\]

MULTIPLICATION of TWO SIGNALS!!
Electronics are Boring if you only have Linear Devices

• Linear Devices (resistors, inductors, capacitors) will expand at most to something like this:
  \[ f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2}(x - c)^2 + \cdots \]
  
• No second order term
• Non-linear devices will have that and more potentially
  \[ f(x) = f(c) + f'(c)(x - c) + \frac{f''(c)}{2}(x - c)^2 + \cdots \]
  
Gives us multiplication!

That Diode We Were Talking About!

• Its voltage current behavior is approx. exponential
• Infinite Taylor Series
• We can use it to modulate and demodulate signals for transmission and reception and do lots of things!

Anywhere in the Universe where Computation Occurs...you have non-linearities.

• Brain/Neurons
• Artificial Neural Nets
• Electronics (Vacuum tubes were first reliable one)
• Etc...

Are Non-Linear Things Elsewhere?

• Metal into a Semiconductor leads to a point-contact junction (acts like a diode)
• Cat Whisker Detectors

• Some of the first commercial detector kits used these instead
Crystal Radio (~1900)

- Lead Sulfide
- Make a diode from metal-semiconductor junction
- Recover radio signal
- No amplification so headsets only