Switching Supplies I

Buck Converters

Voltagess

• It is advantageous to transfer power at high voltages

If power is conveyed with high currents (small voltages), need to minimize power loss by shrinking \( R_{PL} \)...means you need to make low-resistance wires...uses lots of copper...needs lots of money!

\[
p_R = i_R^2 R_{PL}
\]

If power is conveyed with high voltages (small currents), power loss isn’t as big of a concern so we don’t need to make \( R_{PL} \) crazy small!

\[
p_R = \frac{v_R^2}{R_{PL}}
\]

• High voltage, minimizes loss mechanisms in transfer medium (the wires)
Need to scale voltages down

Equivalent to 120V DC out (is actually AC)

Puts out 5V! Is so small yet flavorful!

Historically the answer was transformers!

- Transformers can scale one (AC) voltage up or down to another AC voltage at high efficiency.
- Downside of transformers is they need to be made of iron (heavy) :/
- Don’t scale super well :/
- Work with AC only :/

We convert AC to DC using diodes. We’ll start to talk about them on Thursday

https://www.electronics-tutorials.ws/transformer/transformer-basics.html
Linear Regulator

- Add a resistive-like element in series with load to cook off excess power (voltage divider)

![Linear Regulator Circuit Diagram]

- Downside of course is you waste lots of energy in that regulator resistor :/ 

Switching Supplies

- What are used in almost all modern power supplies
- Utilize some nifty second order circuits to scale up or down voltages
- Scale down (High DC voltage to lower DC voltage):
  - Buck converter
- Scale up (Low DC voltage to higher DC voltage):
  - Boost converter
- Do both (one DC voltage to another DC voltage):
  - Buck-Boost (within reason)
- Don’t need transformers (big ones anyways) so can be made lighter and smaller for a given power capability!
Buck Converter

- Takes an input voltage and produces a lower output voltage

FET and diode will be covered in coming lectures!

Buck Converter

- The FET and diode act like complementary switches that toggle very quickly

- When $S_1$ is closed, $S_2$ is open
- When $S_1$ is open, $S_2$ is closed
Buck Converter Operation

- Phase 1: When $S_1$ is closed, $S_2$ is open

\[ i_L(t) = \int v_{IN} - v_{OUT} \]

- Current through the inductor builds up!

- Phase 2: When $S_1$ is open, $S_2$ is closed

\[ i_L(t) = \int 0 - v_{OUT} + i_L(t_{sw1}) \]

- Current through the inductor starts to decay away (but stays positive since it started positive)
Buck Converter Operation

• Phase 1: When \( S_1 \) is closed, \( S_2 \) is open

\[
i_L(t) = \int v_{IN} - v_{OUT} + i_L(t_{sw2})
\]

• Current through the inductor starts to decay away (but stays positive since it started positive)

---

Simplified Circuit

\[ v_S, v_{IN} \]

\[ L, C, R_L \]

\[ v_{OUT} \]
Sinusoidal Steady-State/Impedance Approach

That Signal...

Remember $D$ is the duty cycle (varying from 0 to 1)

- That input signal can be decomposed into a bunch of sinusoids thanks to Fourier Theory

$$v_s(t) = DV_{IN} + \sum_{n=1}^{\infty} \frac{2V_{IN} \sin(n\pi D)}{n\pi} \cos(n2\pi f_s t)$$

$$f_s = \frac{1}{T}$$

Don’t worry about how to derive this one, just make piece with fact that it can be derived

Switching frequency
That Signal...

Remember $D$ is the duty cycle (varying from 0 to 1)

$v_s(t) = DV_{IN} + \sum_{n=1}^{\infty} \frac{2V_{IN}}{n\pi} \sin(n\pi D) \cos(n2\pi f_s t)$

$f_s = \frac{1}{T}$

Switching frequency

Higher frequency terms have smaller and smaller amplitudes, and therefore have less of an impact on the overall signal

@ $D = 0.5$ for example...

$v_s(t) = \frac{V_{IN}}{2} + \frac{2V_{IN}}{\pi} \cos(2\pi f_s t) + \frac{2V_{IN}}{3\pi} \cos(6\pi f_s t) + \frac{2V_{IN}}{5\pi} \cos(10\pi f_s t) + \cdots$

Transfer Function?

• How does this circuit respond to different frequencies then?

1.) $v_s(t) = V_s \cos(\omega t + \phi_s)$

2.) $v_s(t) = V_s(e^{j(\omega t + \phi_s)} + e^{-j(\omega t + \phi_s)})$

3.) $v_s(t) = \text{Re}\{V_s e^{j(\omega t + \phi_s)}\} = \text{Re}\{\tilde{V}_s e^{j\omega t}\}$

( $\tilde{V}_s = V_s e^{j\phi_s}$ )

4.) $\tilde{V}_s e^{j\omega t}$ will lead to $\tilde{V}_o e^{j\omega t}$

( $\tilde{V}_o = V_o e^{j\phi_o}$ )

5.) Solve for $\tilde{V}_o$ based on $\tilde{V}_s$ (transfer function) (ignore $e^{j\omega t}$ since it is common to all terms)

6.) Add back in $e^{j\omega t}$: $\tilde{V}_o e^{j\omega t}$

7.) $\text{Re}\{V_o e^{j(\omega t + \phi_o)}\} = \text{Re}\{V_o e^{j(\omega t + \phi)}\}$

8.) $\text{Re}\{V_o e^{j(\omega t + \phi)}\} = V_o (e^{j(\omega t + \phi)} + e^{-j(\omega t + \phi)})$

9.) $v_o(t) = V_o \cos(\omega t + \phi_o)$

From magnitude of TF

From angle of TF
\[ \tilde{V}_0 \]

\[ \frac{\tilde{V}_0}{\tilde{V}_s} = \frac{Z_b}{Z_+ + Z_b} = \frac{R}{1 + j\omega RC} \]

Get it into a form so the denominator is \( \omega_0^2 + j\omega \alpha - \omega^2 \)

\[ \frac{\tilde{V}_0}{\tilde{V}_s} = \frac{R}{-\omega^2 RLC + j\omega L + j\omega HR} = \frac{1}{-\omega^2 LC + j\omega L + \frac{1}{RC}} = \frac{-\omega^2 LC + j\omega L + \frac{1}{RC}}{\phi} \]

Magnitude?

\[ \frac{|\tilde{V}_0|}{|\tilde{V}_s|} = \frac{1}{\sqrt{(\frac{1}{LC})^2 + (\frac{-\omega^2}{RC})^2}} \]

Phase:

\[ \phi = \tan \left( \frac{\omega}{\frac{1}{LC}} \right) \]

\[ = \tan \left( \frac{1}{\frac{1}{LC} - \omega^2} \right) \]

\[ \angle \frac{\tilde{V}_0}{\tilde{V}_s} = -\tan \left( \frac{\omega}{\frac{1}{LC} - \omega^2} \right) \]
So the behavior of the magnitude at the natural undamped resonant frequency of the filter is based on the ratio of $R$ to the characteristic impedance.

 characterize impedance of an RLC circuit

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$Q = \frac{\omega_0}{2\alpha} = \frac{\frac{1}{R+\frac{1}{LC}}}{\omega_0}$$

$$Q = \frac{R}{2\omega_0}$$

So the behavior of the magnitude at the natural undamped resonant frequency of the filter is based on the ratio of the $R$ to the characteristic impedance.

So what is that value at $\omega_0$ for various "tunings" of the circuit? For this circuit:

1. $\alpha = \frac{1}{2\omega_0}$, $\omega_0 = \frac{1}{R \omega_0}$

2. $\alpha \ll \omega_0$ "underdamped" $\frac{1}{2\omega_0} \ll \frac{1}{\sqrt{LC}} \ll R$

3. $\alpha = \omega_0$ "critically damped" $\frac{1}{2\omega_0} = \frac{1}{\sqrt{LC}} \rightarrow \frac{1}{2\sqrt{\frac{L}{C}}} = R$

4. $\alpha \gg \omega_0$ "overdamped" $\frac{1}{2\omega_0} \gg \frac{1}{\sqrt{LC}} \gg R$
Underdamped \( \alpha < \omega_o \)

@\(\omega_o\): \(\frac{R}{Z_o}\)

Critically Damped \( \alpha = \omega_o \)

@\(\omega_o\): \(\frac{R}{Z_o}\)
OverDamped \[ \alpha > \omega_0 \]

\[ R = Z_0 \text{ (technically slightly underdamped)} \]

\[ 2\alpha = \omega_0 \]
The Frequency Response and Time Response are Related!

- **Three cases**

  \[ \alpha < \omega_0 \]  
  **Underdamped**
  \[
  s_1 = -\omega + j\omega_0 \\
  s_2 = -\omega - j\omega_0 
  \]
  \[
  v_{ch} = e^{-\alpha t} \left( A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t} \right) 
  \]
  \[
  v_{ch} = e^{-\alpha t} (A_3 \cos(\omega_d t) + A_4 \sin(\omega_d t)) 
  \]
  \[
  v_{ch} = A_5 e^{-\alpha t} \cos(\omega_d t + \phi_0) 
  \]

  \[
  Q = \frac{\omega_0}{2\alpha} 
  \]
  Quality factor

  \[ \alpha > \omega_0 \]  
  **Overdamped**
  \[
  s_1 = -\alpha + \sqrt{\alpha^2 - \omega^2} \\
  s_2 = -\alpha - \sqrt{\alpha^2 - \omega^2} 
  \]
  \[
  v_{ch} = A_1 e^{z_1 t} + A_2 e^{z_2 t} 
  \]

  \[ \alpha = \omega_0 \]  
  **Critically damped**
  \[
  s_1 = s_2 = -\alpha 
  \]
  \[
  v_{ch} = A_1 e^{-\alpha t} + A_2 e^{-\alpha t} 
  \]

Underdamped  \[ \alpha < \omega_0 \]

Decay of envelope reveals \( \alpha \)
Period of oscillations reveal \( \omega_d \)
\( \omega_d \) and \( \alpha \) can lead to \( \omega_0 \)

\[ Q = \frac{\omega_0}{2\alpha} \ldots \] So \( Q \) starts to get big
Overdamped

\[ \alpha > \omega_o \]

Sluggish response in time
(slower than critically damped)

Frequency response is also more gradual

Critically Damped

\[ \alpha = \omega_o \]

Faster response in time,
but no overshoot/oscillations

Frequency response goes from flat
to steep downwards pretty quickly
\[ R = Z_o \] (slightly underdamped)

\[ Q = \frac{\omega_o}{2\alpha} = \frac{R}{Z_o} = 1 \]

What frequency should we run our Buck Converter at?

\[ v_s(t) = \frac{V_{IN}}{2} + \frac{2V_{IN}}{\pi} \cos(2\pi f_s t) + \frac{2V_{IN}}{3\pi} \cos(6\pi f_s t) + \frac{2V_{IN}}{5\pi} \cos(10\pi f_s t) + \cdots \]

We want a steady output voltage for our downstream electronics!
What frequency should we run our Buck Converter at?

As a result

• That way:

\[ v_s(t) = \frac{V_{IN}}{2} + \frac{2V_{IN}}{\pi} \cos(2\pi f_s t) + \frac{2V_{IN}}{3\pi} \cos(6\pi f_s t) + \frac{2V_{IN}}{5\pi} \cos(10\pi f_s t) + \cdots \]

\[ v_s(t) \approx \frac{V_{IN}}{2} \]
Some ripple:

\[ v_s(t) = \frac{V_{IN}}{2} + \frac{2V_{IN}}{\pi} \cos(2\pi f_s t) + \frac{2V_{IN}}{3\pi} \cos(6\pi f_s t) + \frac{2V_{IN}}{5\pi} \cos(10\pi f_s t) + \cdots \]

- Let’s consider the first sinusoidal term:
  - \( f_s = 40 \text{ kHz} = 80\pi \text{ krad/s} \)
  - \( \omega_o = 2.6 \text{ krad/s} \)
- Amplitude of that first fundamental:

\[
\frac{2V_{IN}}{\pi} \cos(2\pi f_s t) \quad \omega \gg \omega_o \quad \text{so} \quad H(j\omega) \approx \frac{1/LC}{\omega^2}
\]

@ \( 80\pi \text{ krad/s} \): 

\[
H(j\omega) \approx \frac{1/LC}{\omega^2} = \frac{V_{IN}}{\frac{f^2_s}{2}2LC}
\]