Nov 5, 2019

1. Modeling the world
   Why?
   1. Allow us to design/analyze transducers
   2. Gain intuition into other domains

2. Translational mechanics
   How?
   Use language of circuits

3. Translational mechanics
   \[ F = kx \]

For linear spring:
\[ F = kx \]
Store mechanical pot. energy

3. Circuit variables
   Electric circuits: \( v \rightarrow i \rightarrow P = v i \) voltage

Spring: \( w = F \rightarrow P = F \rightarrow v \) velocity

\[ F \rightarrow v \quad \text{or} \quad F \rightarrow L \]
Both are used

Magnetic spring:
\[ v \rightarrow F, \quad F \rightarrow i, \quad i \rightarrow v, \quad v \rightarrow F \]
11/5/19

**Other components**

- **Mass**
  
  \[ F = \frac{m}{m} \frac{dv}{dt} \Rightarrow v = \frac{F}{m} \]

- **Damper**
  
  \[ F = bV \quad \text{[N-s/m]} \]

- **Sources**
  
  \[ + \quad V = V_0 \quad - \quad F = F_0 \]

**Connections**

- Share common velocity \( \Rightarrow \) parallel
- Share common force \( \Rightarrow \) series

\[ \text{KVL: } \sum V = 0 \]

\[ \text{KCL: } \sum F = 0 \]

**Skydiver**

\[ b = \text{Air friction} \]

\[ F = mg \]

**Spring-mass**

\[ mg = \frac{1}{b} + \frac{m}{dt} \]

\[ mg = bv + \frac{m}{dt} \leq 2^{nd} \text{ law} \]

\[ \omega_0 = \sqrt{\frac{k}{m}} \]
Accelerometer

- Analog Devices Accelerometer
- ADXL150
- Acceleration $\rightarrow$ changes gap $\rightarrow$ changes capacitance $\rightarrow$ electrical output

Questions:
- How quickly does this sensor respond?
- How big is the signal?

Multi-domain systems are critical part of renewable energy systems
Mass balance

• Converts change in mass \( \rightarrow \) change in electrical frequency
Modeling transducers

Free-body diagrams

Mechanical ODEs

Transducer

Transducer ODEs

Electrical circuits

Electrical ODEs

System ODEs

THIS IS AN RC CIRCUIT
A NEW ANALOGY BETWEEN MECHANICAL AND ELECTRICAL SYSTEMS

By F. A. Firestone
University of Michigan

ABSTRACT

By considering each mass in a linear mechanical system as having two terminals, one fixed in the mass and one fixed to the frame of reference, every linear mechanical system is reduced to a multiplicity of closed mechanical circuits to which force and velocity relations similar to Kirchhoff’s laws, may be applied. The conventional mechanical-electrical analogy is derived from the similarity of the equations \( v = f/s \) and \( I = E/Z \). It is incomplete in the following respects which lead to difficulty in its application:

1. There is a lack of analogy in the use of the words “through” and “across” which indicates a fundamental difference in the nature of the analogous quantities.

Journal of the Acoustical Society of America, 1953

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**Force \( \rightarrow \) voltage**

**Velocity \( \rightarrow \) current**

- Direct analogy
- Impedance analogy
- Pros
  - Potential energy = potential energy
- Cons
  - Topology is difficult

**Force \( \rightarrow \) current**

**Velocity \( \rightarrow \) voltage**

- Indirect analogy
- Mobility analogy
- Admittance analogy
- Pros
  - Topology is maintained
- Cons
  - P.E \( \rightarrow \) K.E and vice versa

*This is what we’ll use*

BEWARE OF LITERATURE AND TEXTBOOKS!
### Capacitors

<table>
<thead>
<tr>
<th>Element</th>
<th>Constitutive equation</th>
<th>Elemental equation</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational mass</td>
<td>( p = m \dot{v} )</td>
<td>( F = m \ddot{v} )</td>
<td>( E = \frac{1}{2}mv^2 )</td>
</tr>
<tr>
<td>Rotational inertia</td>
<td>( h = J \omega )</td>
<td>( T = J \dot{\omega} )</td>
<td>( E = \frac{1}{2}I\omega^2 )</td>
</tr>
<tr>
<td>Electrical capacitance</td>
<td>( q = Cv )</td>
<td>( i = \frac{dv}{dt} )</td>
<td>( E = \frac{1}{2}Cv^2 )</td>
</tr>
<tr>
<td>Fluid capacitance</td>
<td>( V = C_f \rho )</td>
<td>( Q = C_f \frac{d\rho}{dt} )</td>
<td>( E = \frac{1}{2}C_f \rho^2 )</td>
</tr>
<tr>
<td>Thermal capacitance</td>
<td>( H = C_{T}T )</td>
<td>( q = C_{T} \frac{dT}{dt} )</td>
<td>( E = C_{T}T )</td>
</tr>
</tbody>
</table>

### Inductors

<table>
<thead>
<tr>
<th>Element</th>
<th>Constitutive equation</th>
<th>Elemental equation</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational spring</td>
<td>( x = \frac{1}{K} \dot{F} )</td>
<td>( v = \frac{1}{K} \ddot{F} )</td>
<td>( E = \frac{1}{2}Kv^2 )</td>
</tr>
<tr>
<td>Torsional spring</td>
<td>( \Theta = \frac{1}{K_t} T )</td>
<td>( \Omega = \frac{1}{K_t} \dot{T} )</td>
<td>( E = \frac{1}{2}K_t\theta^2 )</td>
</tr>
<tr>
<td>Electrical inductance</td>
<td>( \lambda = L \dot{i} )</td>
<td>( v = L \frac{di}{dt} )</td>
<td>( E = \frac{1}{2}L\dot{i}^2 )</td>
</tr>
<tr>
<td>Fluid inductance</td>
<td>( \Gamma = i_f \dot{Q} )</td>
<td>( P = i_f \frac{dQ}{dt} )</td>
<td>( E = \frac{1}{2}i_f Q^2 )</td>
</tr>
</tbody>
</table>

### Resistors

<table>
<thead>
<tr>
<th>Element</th>
<th>Elemental equations</th>
<th>Power dissipated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translational damper</td>
<td>( F = B_e )</td>
<td>( P = BV_e^2 )</td>
</tr>
<tr>
<td>Rotational damper</td>
<td>( T = B_r \Omega )</td>
<td>( P = BV_r \Omega^2 )</td>
</tr>
<tr>
<td>Electrical resistance</td>
<td>( i = \frac{v}{R} )</td>
<td>( P = \frac{1}{2}v^2 )</td>
</tr>
<tr>
<td>Fluid resistance</td>
<td>( Q = \frac{1}{R_f}P )</td>
<td>( P = \frac{1}{2}P^2 )</td>
</tr>
<tr>
<td>Thermal resistance</td>
<td>( q = \frac{1}{R_t}T )</td>
<td>( P = R_t q^2 )</td>
</tr>
</tbody>
</table>

### Sources

<table>
<thead>
<tr>
<th>Energy Domain</th>
<th>Across-variable source</th>
<th>Through-variable source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanical translational</td>
<td>Velocity source ( V_e(t) )</td>
<td>force source ( F_e(t) )</td>
</tr>
<tr>
<td>Mechanical rotational</td>
<td>Angular velocity source ( \Omega_e(t) )</td>
<td>Torque source ( T_e(t) )</td>
</tr>
<tr>
<td>Electrical</td>
<td>Voltage source ( V_e(t) )</td>
<td>current source ( I_e(t) )</td>
</tr>
<tr>
<td>Fluid</td>
<td>Pressure source ( P_e(t) )</td>
<td>Flow source ( Q_e(t) )</td>
</tr>
<tr>
<td>Thermal</td>
<td>Temperature source ( T_e(t) )</td>
<td>Heat flow source ( Q_e(t) )</td>
</tr>
</tbody>
</table>
Moth energy harvester mechanical system

- Spring
- Metglas®/Steel Magnetic Core
- NdBFe/SmCo Permanent Magnets
- Printed-circuit windings

- Mechanical Resonator
- Generator

- Power Electronics

- Moth thorax

1 mm, 25 Hz
1 N, 40 mW

~4 mm

Power

Electronics

~1 mW

Moth energy harvester mechanical system

- Mass
- Damping
- Spring

- Shaker

K B

xH, vH

xS, vS
Moth energy harvester mechanical system

\[ V_H = \frac{Z_{kb}}{Z_{kb} + \frac{1}{j\omega M}} V_S \]

\[ Z_{kb} = \frac{j\omega}{k} \parallel \frac{1}{b} \]

\[ Z_{kb} = \frac{j\omega k b}{j\omega k b + k + j\omega b} \]

\[ \frac{j\omega}{k + j\omega b} \]

\[ V_H = \frac{V_S}{k + j\omega b + \frac{1}{j\omega M}} \]

\[ V_H = \frac{(j\omega)^2}{(j\omega)^2 + (j\omega)\frac{b}{M} + \frac{k}{M}} V_S \]

\[ (j\omega)^2 + (j\omega)2\alpha + \omega_0^2 \]

\[ (j\omega)^2 + (j\omega)\frac{\omega_0}{Q} + \omega_0^2 \]

\[ \frac{|V_H/V_S|}{0.01\omega_0, 0.1\omega_0, \omega_0, 10\omega_0} \]

\[ \text{Frequency (rad/s)} \]