Lecture 10 6.002
October 8th, 2019

We previously dealt with op amp and we came up with a general first attempt at a model of them which said that this symbol was actually this:

\[ V_{\text{out}} = \frac{-R_2}{R_1} V_{\text{in}} \]

where \( A \) is very large

Few golden rules:
1. \( i_+ = 0 \) and \( i_- = 0 \)
2. \( i_o \) is unrestricted (output looks like a voltage source so it can source/sink any current)
3. And also... when held in negative feedback ONLY (meaning there is a path for current to flow from \( v^+ \) to \( v^- \)), the voltages at \( v^+ \) and \( v^- \) will be held together at the same value.

We could then put this op amp in lots of different configurations to get really useful circuits, but one thing that was spoken about is that you really can’t connect the input to the “+” input... (the non-inverting input)... instead you can only connect it (either directly or through a component) to the “-” input. So...

Mathematically if you actually tried to drop the dependent circuit equivalent up above you’d find out, much to your concern, that both circuits solve out ok... don’t believe me? Let’s try:

\[
\begin{align*}
  v_o &= \frac{-R_2}{R_1} v_{\text{in}} \\
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\end{align*}
\]

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\]

This one happens in real life... this one doesn’t... our model isn’t good!!!
We need to develop a new model and thankfully we've built up the necessary tools in the last few weeks!

Time constants!

Our new model looks like the following:

That's right! It has a capacitor in it! And not just a capacitor, but a resistor and a voltage source. In fact, it is our old friend the series RC circuit! Hello, old friend!

What does this mean? It means that our op amp's behavior will be a function of time and the output voltage is now slightly "decoupled" from the v+ and v- voltages. So we should drop this more complicated model into our original two circuits up above and see what they give us!
If $v(t)$ is changing slowly, then $v_0(\infty)$ can be approximated as:

$$v_0(\infty) = -\frac{R_2}{R_1} v_i(t)$$

$$v_0(t) = -\frac{R_2}{R_1} v_i + \left(v_0(0) - \frac{R_2}{R_1} v_i\right) e^{-t/\tau}$$

A is usually very large so this time constant is usually very small...so an op amp usually converges to its steady-state value very quickly...so quickly that we normally don’t see it! But it does happen!

So in summary, the ability of the + and - inputs to influence the output is not immediate, but rather is decoupled by a first-order-like behavior which we can model as a RC network. In general, the time dynamic of the op amp (its $\tau$) is very, very fast and ideally much faster than the rest of the circuit so that to the rest of the circuit the op amp looks like it just appears as its final values (Which we have solved before). Here’s a six-frame step-by-step of how an op amp in the “buffer” format (note there’s negative feedback here!!!) responds to a step of voltage going from 0V to 5V!

1. The op amp is in negative feedback with an input source
2. The input source starts at 0V...the + input is therefore at 0V...the output and - input are also at 0V!
3. The input steps to 5V immediately changing the + input on op amp. The voltage ($v_+ - v_-$) now starts charging the internal capacitor of the op amp which shows up in output voltage!
4. A small time later, the capacitor has built up voltage, meaning output voltage has built up!...that voltage is fed back however, so $v_-$ has been brought towards $v_+$. The capacitor is still charging so the output voltage is still growing
5. A tiny bit later output has increase even more, and output has grown and $v_-$ has been brought even closer to $v_+!$
6. System eventually reaches a equilibrium point with $v_-$ approximately equal to $v_+$, and the output voltage being about $A(v_+ - v_-)$

Normally the op amp gets to step 6 so quickly and $A$ is so large that we just say when in negative feedback $v_+$ will equal $v_-$. This is generally a fine assumption to make unless stated otherwise.
So in our refined model of Op Amps:

- The input current to the + and - inputs (the non-inverting and inverting inputs) is always 0A.
- The output of the op amp looks like a voltage source which produces a value based on:
  \[ v_o = A \left( v_+ - v_- \right) \]

- The op amp generates this voltage using first order dynamics, but usually these dynamics are so fast (tau is so small) that we approximate them as appearing instantly. However, the presence of this time dynamic explains why we can't put op amps in positive feedback and expect them to act the same.
- When in negative feedback, the inverting input (v-) will be brought very close to the value of the non-inverting input (v+). It will do this using the same time dynamics as it generates its output voltage. As a result, because the two voltages are so close and because they equilibrate so fast, we often say v+ = v- for easy solving. This is generally a good approximation to make unless otherwise stated.

In 6.002 when we put op amps in negative feedback, they let us build helpful amplifier circuits that can scale signals up or down, add or subtract them, and do other various things. All the circuits discussed two weeks ago with op amps all had negative feedback in common (some had a little bit of positive feedback such as that one that acted like a current source, but were majority negative). These were "stable" circuits that "converged" on finite output values.

Op amps in "open loop" or in positive feedback also have uses, however...just a different set of uses. We usually call op amps in these situations Comparators (** note below)

For example what would the circuit below do?

\[ v_o = A \left( v_+ - v_- \right) \]

\[ A \gg \text{very large (100,000 or more)} \]

\[ v_o = A \left( v_1 - v_2 \right) \]

\[ \text{extremely steep (almost vertical) because } A \text{ large.} \]

Is that how it will "really" act in real life? What other thing is going on with these op amps/devices that we sometimes ignore?

Power Supply Limitations?

Yes. Power supply limitations.

**Op amps and Comparators are somewhat different devices. In real-life you buy dedicated op amps and dedicated comparators, but theoretically op amps can either serve as negative-feedback gain blocks or comparators. It is kind of like how distance runners and sprinters are both "running athletes" and both could do the other's job, but not as well so there's some specializations that go beyond the scope of 6.002 we need not worry about...for now.**
All op amps exist within the constraints of their power supply. They can only output a voltage that exists between \(+VS\) and \(-VS\)....this means the output of our open loop comparator circuit really looks like the following:

So this circuit has an output equation of the following:

\[
V_o = \begin{cases} 
+V_S & \text{if } V_1 > V_2 \\
-V_S & \text{if } V_1 < V_2 
\end{cases}
\]

It is almost as if this circuit is doing something....what's the word I'm looking for....it is looking at one value...and another and deciding based on that...comp...compare? Ahh yes it is comparing voltages. That's why it was named a comparator. All is right with the world.

We'll often setup comparators so they have a "reference" voltage and an "input" voltage, and depending on which side you put which voltage you can get what's called either a "positive" or "negative" comparator.
Consider the example of a comparator here where the reference voltage is on the non-inverting input and it is set by a voltage divider...an input signal shown to the left \((v_I)\) is fed in to the inverting input...what would the output be?

We use comparators all the time when we need to “threshold” or “quantize” analog voltages. When my dad used to tell me that he’s “had it up to here with [my] behavior” he was being a comparator...my analog behavior signal was thresholded into a punishment signal and I would lose dialup internet privileges.

If we return to our equation for one of the comparators, however a potential problem/hole might appear, though....

What happens if \(v_{IN} = v_{REF}\)? Or even what if they are super-close by.

We’re relying on \(A\) to be very large to quickly swing from one extreme to another, but even \(A\) has its limits

This could actually be a problem. The system becomes susceptible to noise and other messy things...

The solution is to bring back positive feedback into our circuit to give us an awesome benefit:

- It will introduce hysteresis (moving the effective threshold in response to where the output is) which will let it be resistant to noise
Consider this circuit:

Positive comparator with hysteresis.

If we start with $v_o = +V_s$ what must $v_{IN}$ fall below to switch $v_o$ to $-V_s$?

we need $v_+ < v_{REF}$

If we start with $v_o = -V_s$ what must $v_{IN}$ rise above to switch $v_o$ to $+V_s$?

we need $v_+ > v_{REF}$

The result of all of this is that our relationship between $v_{OUT}$ and $v_{IN}$ is now based on a hysteresis curve.

If we call $-V_s$ LOW and $+V_s$ HIGH

In order to switch from LOW to HIGH we need to pass a higher threshold than what we to drop below when falling from HIGH to LOW...

Basically as soon as the output changes, the input requirements for another output flip change so that it they are further away. This makes it way more noise resistant!!!
Now the same circuit with hysteresis...

Noisy Input signal

Comparator output with no hysteresis

Noise is reduced.

Comparator output with hysteresis

$U_{REF}$ shifts to cut out noise.
• If the circuit starts with $V_0 = 5V$ and the capacitor uncharged, the capacitor will start charging through $R$.
• The voltage at $v+$ is 2.5V.
• When the cap voltage passes 2.5V the circuit flips...Now the output is 0V and $v+$ becomes -2.5V.
• The capacitor tries to charge towards that voltage...eventually gets there...and the circuit flips again!
• And again.
• And again.
• We have an oscillator where the frequency is based on the $R$ and $C$ you pick!