Last week we took a first look at capacitors and inductors. These are components that store energy (rather consume or supply it). Because they store energy, they enforce an interesting set of requirements upon us when solving circuits in time.

Capacitors: Store their energy based on their voltage. They have this device law. They enforce the rule that the voltage across them MUST! ALWAYS! BE! CONTINUOUS!

\[
\lim_{t \to t^-} v_c(t) = \lim_{t \to t^+} v_c(t)
\]

Otherwise this derivative would not exist/be undefined...no good!

Inductors: Store their energy based on the current through them. They have this device law. They enforce the rule that the current through them MUST ALWAYS BE CONTINUOUS!!!

\[
\lim_{t \to t^-} i_L(t) = \lim_{t \to t^+} i_L(t)
\]

Otherwise this derivative would be undefined/not exist...which is not good!

Then we briefly solved a circuit where there was a voltage source, a resistor, and a capacitor:

\[
\begin{aligned}
\text{Example:} & \quad \text{If} \quad v_h(t) = \begin{cases} 0V \quad t < 0, \\ 5V \quad t \geq 0 \end{cases} \\
\text{Then rearrange...} & \quad v_i(t) = v_c(t) + RC \frac{dv_c(t)}{dt} \\
\text{This is an ODE} & \quad \text{with general solution:} \quad v_c(t) = A_1 + A_2 e^{-t/RC} \\
\text{where} A_1 \text{ and } A_2 \text{ come from initial and final conditions of the circuit.} & \quad v_c(0) = 0V, \quad v_c(t) = 5V - 5e^{-t/RC} \\
\text{Nodal analysis here:} & \quad \frac{v_i(t) - v_c(t)}{R} = C \frac{dv_c(t)}{dt}
\end{aligned}
\]
Let's dwell on this simple circuit a bit more... It might seem very specific, but that voltage source and resistor also take the same shape/topology as a Thevenin circuit (remember)...and since any circuit can be converted into a Thevenin equivalent, this circuit is very useful in understanding how a capacitor plays with other circuits in general!

So it has a solution of the following:

\[ v_c(t) = v_c(\infty) + (v_c(0^+) - v_c(\infty))e^{-t/RC} \]

The first order dynamics will dictate how the circuit evolves going from one "event" in time to another where an "event" is some change in the circuit that occurs. An event could be:

- The turning on/off of a voltage source
- The turning on/off of a current source
- The closing/opening of a switch...which could:
  - introduce an additional device
  - de-introduce (remove) an additional device
- Change the value of a device (voltage source goes from 5V to 3V...current source goes from 1A to 2A, etc...)

For convenience let's think of our event as happening at \( t=0 \) (it doesn't have to by any means, but it makes our math easier if we just say that)

To figure out what our circuit's initial and final values must be, we need to think of what the circuit looks like just before and a long time after the event happens!

For example:

For \( t < 0 \):
\[ v_c(t) = \begin{cases} 0 \text{V} & \text{for} \ t < 0 \\ 5 \text{V} & \text{for} \ t \geq 0 \end{cases} \]

Before \( t = 0 \):

After \( t = 0 \):

\[ 5 \text{V} \]
To determine what the circuit looks like right before the change and right after we need to know if we can make any assumptions.

If it was stated that the circuit had been allowed to sit for a long time before t=0, that means the circuit had time to equilibriate. This allows us to think about the capacitor in an extreme case.

*If the circuit is not longer changing (has reached some sort of equilibrium, that means the derivative of any voltage across the capacitor will be 0...that means the current through the capacitor will be 0. This means that in a circuit in equilibrium (one that has been allowed to sit for a long time), a capacitor will “look” like an open circuit!*

![](image1.png)

We can use this to our advantage! It allows us to make approximations of the circuit and then solve those relatively easily...

So in our example up above...if circuit has been allowed to sit a long time before the "event" happens it'll look like this:

![Resistor Circuit](image2.png)

We can solve this circuit (just a resistor) to figure out what the voltage was right before the transition!!!

*this is just a resistor with no sources, so solving it is easy!*

\[ v_c(0^-) = 0V \]

where 0^- means just before the event (t=0)

Now how do we use this value to help us solve our circuit? As has already been said, a capacitor forces the voltage across it to always be continuous (not be discontinuous)

As a result the voltage across the capacitor ‘right’ before the event \( v_c(0^-) \) and the voltage right after the event \( v_c(0^+) \) must be the same!!!

\[ as \ a \ result: \ v_c(0^-) = v_c(0^+) \]

\[ \text{initial condition} \]
We can do the same sort of thing if finding what our circuit's desired final condition is...analyze the circuit after the event in equilibrium...

Relatively easy to solve...we can see that:

\[ V_c(\infty) = 5V \]

We now have the initial and final conditions!

\[ V_c(0^+) = 0V \quad V_c(\infty) = 5V \]

We therefore know where our system's first order response will start and where it will end!

The initial conditions don't have to always be 0 btw!!! We'll look at an example in a little bit with that situation!

But how will it change??? This is where the RC time constant comes into play (\( \tau \))

\[ V_c(t) = V_c(\infty) + (V_c(0^+) - V_c(\infty))e^{-t/\tau} \]

The dynamics at which the system moves evolves from the initial condition to the final condition will always be first order...but the rate at which they do that is based on the RC time constant.

Intuitively...if \( R \) is bigger, it let's less current through, meaning it takes longer to charge up (or down) the capacitor.

If \( C \) is bigger...the capacitor (remember \( Q=CV \)) needs more charge to build up a voltage...so it will take longer to charge up (or down) the capacitor.

If \( R \) is smaller...let's more current through...takes less time to charge up or down!

If \( C \) is smaller...the capacitor (remember \( Q = CV \)) needs less time to build up the charge necessary for a voltage!

SO....let's do an example!!!....next page:
We have the circuit to the right. The switch is open for a very long time. At \( t=0 \), the switch is closed. Provide an expression and plot of how the capacitor voltage varies over time...

First think about what is the circuit doing on both sides of "the event"...where "the event" refers to the switch closing:

It sits in these two states a very long time before and after the switch closing so in both situations we can assume that the circuit has reached an equilibrium (and all time-dependent terms have decayed away). As a result the capacitor will look like an "open" in those situations:

If we solve these equivalent steady-state circuits we can figure out exactly what the voltage across the capacitor will be right before the event!

And as a lot of time passes! \( t \rightarrow \infty \):

\[
\nu_c(0^-) = 5V \\
\nu_c(\infty) = 2.5V
\]

The capacitor forces the voltage across it to be continuous (cannot change instantaneously otherwise the universe would break down and we don't want that)...so this means that:

\[
\nu_c(0^+) = 5V
\]

We now have our initial and final conditions...and we just need to figure out our time dynamics and that's going to be based on our resistor and capacitor values!...the question becomes how do those parts all work together...which resistor (or both do we use??). Well think about the circuit from the point of view of the capacitor! What does the capacitor "electrically" see?
We can generate the Thevenin Equivalent of the circuit as seen from the capacitors point of view in order to make a more easily solved circuit (in fact the circuit we'll generate will already have been solved in the general sense earlier!!!)

Now our time dynamics are just gonna be dictated by this circuit:

And we already know that this circuit has the following solution:

\[ v_c(t) = v_c(\infty) + (v_c(0) - v_c(\infty))e^{-t/RC} \]

where R is 500 Ohms and C is 10 uF and the initial val is 5V and the final val is 2.5V so!!!

\[ v_c(t) = 2.5 + 2.5 e^{-t/0.005} \]

So...

\[ 5V \]

\[ e^{-t/0.005} \text{ dynamics} \]

\[ ... 2.5V \]

\[ t=0 \text{ switch thrown} \]
What would happen if some time later (after the switch was closed) it is then opened again?? How will the circuit respond?

Let’s assume for easy solving that this happens a long time after the circuit has come to its new equilibrium of 2.5V....our new event happens at some future time t=t0...

How can we approach this?
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Capacitors
Constrain Voltage

Inductors
Constrain Current

Circuit:

First, right before the event...

At $t=0^-$:
If the circuit has stabilized (values no longer changing)

$\epsilon \gg \tau$

Capacitor looks like an "open"
with $vC(0^-)$ across it. No current flowing through it

Inductor looks like a "short"
with $iL(0^-)$ through it. No voltage is across it!

At $t=0^-$:
If circuit has not stabilized (values still changing)

$\epsilon \gg \tau$

Circuit still evolving...need to solve for $vC(0^-)$ directly!

Circuit still evolving...need to solve for $iL(0^-)$ directly!

Event at $t=0^-$ !!! EVENT AT $t=0^+$ !!! EVENT AT $t=0^+$ !!!

Then right after the event!

At $t=0^+$:
If component starts with non-zero energy in it

At $t=0^+$:
If component starts with zero energy in it ("at rest")

special case of above, 0V cap looks like short:

special case of above, 0 current inductor looks like open:

Finally .... as $t \to +\infty$

Capacitor looks like an "open"
with $vC(0)$ across it. No current flowing through it

Inductor looks like a "short"
with $iL(0)$ through it. No voltage is across it!

At $t=0^+$ and ONLY at $t=0^+$, capacitor will "appear" as a voltage source with a voltage of whatever $vC(t)$ was (voltage right before event)...current through capacitor is free to be whatever is needed (just like a voltage source!!)

At $t=0^+$ and ONLY at $t=0^+$, inductor will "appear" as a current source with a current of whatever $iL(t)$ was (the current right before event)...voltage across the inductor is free to be whatever is needed (just like with a current source!!)

Capacitor looks like an "open"
with $vC(0^-)$ across it. No current flowing through it

Inductor looks like a "short"
with $iL(0^-)$ through it. No voltage is across it!

At $t=0^-$ and ONLY at $t=0^-$, capacitor will "appear" as a voltage source with a voltage of whatever $vC(t)$ was (voltage right before event)...current through capacitor is free to be whatever is needed (just like a voltage source!!)

At $t=0^-$ and ONLY at $t=0^-$, inductor will "appear" as a current source with a current of whatever $iL(t)$ was (the current right before event)...voltage across the inductor is free to be whatever is needed (just like with a current source!!)

Finally .... as $t \to +\infty$

Capacitor looks like an "open"
with $vC(0^-)$ across it. No current flowing through it

Inductor looks like a "short"
with $iL(0^-)$ through it. No voltage is across it!