Lecture 3

12/19

Today

1. SUPERPOSITION

2. THÉVENIN/NORTON

Redo via Superposition

1. Turn off \( V_0 \)

\[
\begin{align*}
\text{overall} & \quad e = \frac{R_2}{R_1 + R_2} V_0 + \frac{R_2 R_I}{R_1 + R_2} I_0 \\
\text{Turn off} & \quad I_0 \\
\text{overall} & \quad e = \frac{R_2}{R_1 + R_2} V_0
\end{align*}
\]

3. THÉVENIN

By superposition

\[
V_x = \sum_{n=1}^{N} a_n V_n + \sum_{m=1}^{M} R_{m} I_m + \sum_{n=1}^{N} R_{m} i_m
\]

Response at port when \( i_m = 0 \)
What circuit has characteristic $V_{TH}$ voltage at $V_X$ when nothing connected externally.

$V_{OC} = \frac{V_X}{I_X=0} = V_{TH}$

$R_{TH}$: resistance looking into the port with all internal sources turned off.

**Model as**

\[
\begin{align*}
V_{TH} &= V_{OC} = e \\
V_{TH} &= \frac{R_1 R_2}{R_1 + R_2} V_0 + \frac{R_1 R_2}{R_1 + R_2} I_0
\end{align*}
\]

**Model as**

\[
R_{TH} = \frac{R_1 R_2}{R_1 + R_2}
\]

\[
R_{TH} = R_N \checkmark
\]

\[
V_{OC} = \frac{V_X}{I_X=0} = V = I_N R_N
\]

For our ex 2:

\[
\begin{align*}
V_{TH} &= V_0 \\
I_N &= \frac{V_{TH}}{R_{TH}} = I_0 + \frac{I_0}{R_1}
\end{align*}
\]

$\Delta$ short circuit current

\[
I_X = -I_{SC} = -I_N \Rightarrow I_N = i_{SC}
\]

\[
R_{TH} = R_N = \frac{R_1 R_2}{R_1 + R_2}
\]

**Norton**

\[
R_N = \frac{V_{OC}}{I_{SC}}
\]

\[
\begin{align*}
V_{OC} &= \frac{V_{TH}}{R_{TH}} = R_{TH} \\
I_{SC} &= \frac{V_{TH}}{R_{TH}}
\end{align*}
\]
Recap

Nodal analysis method

1. **Select a reference node**, called ground, from which all other voltages will be measured. Define its potential to be 0 V.

2. **Label the potentials of the remaining nodes** with respect to the ground node. Any node connected to the ground node through an independent voltage source should be labeled with the voltage of that source. (Also for dependent sources).

3. **Write KCL for each node** that has an unknown node voltage, *immediately* substituting in element law.

4. **Solve the equations** resulting from Step 3 for the unknown node voltages.

5. **Back-solve for the branch voltages and currents.**

Adapted from Lang and Agarwal

Recap

\[
G_1(V_0 - e_1) + G_2(-e_1) + G_3(e_2 - e_1) = 0 \\
G_4(V_0 - e_2) + G_5(-e_2) + G_3(e_1 - e_2) + I_0 = 0
\]

Node voltages are *linear* combinations of independent sources

**ANY** circuit comprised of constant resistors and independent sources is *linear*
Linearity

Homogeneity

\[
\begin{align*}
\text{IF:} & \quad x_1 \rightarrow y_1 \\ & \quad \vdots \\ & \quad x_N \\
\text{THEN:} & \quad \alpha x_1 \rightarrow \alpha y_1 \\ & \quad \vdots \\ & \quad \alpha x_N
\end{align*}
\]

If we scale the inputs, the output scales → Zero input → zero output

Additivity

\[
\begin{align*}
\text{IF:} & \quad x_{1A} \rightarrow y_{1A} \\ & \quad \vdots \\ & \quad x_{NA} \\
\text{THEN:} & \quad x_{1A} + x_{1B} \rightarrow y_{1A} + y_{1B} \\ & \quad \vdots \\ & \quad x_{NA} + x_{NB}
\end{align*}
\]

Sum of the inputs → sum of responses to individual inputs

DAC: digital-to-analog converter

• Used to create analog voltages from digital computers
  • Play music on a speaker
  • Control rotational velocity of a motor
  • Etc.

Important to do the prelab! And bring your kit! And come on time!
Superposition

For any circuit comprised of constant resistors and independent sources $\rightarrow$ LINEAR

**Superposition**

1. Turn off all independent sources but one.
2. Solve for desired response, *aka* node voltage or branch variable.
3. Repeat for remaining independent sources.
4. Total response is sum of individual responses.

---

**Additivity**

\[ V_1 = V_{CC} \rightarrow V_0 = V_{1A} \]
\[ V_2 = 0 \rightarrow V_0 = \]  
\[ V_1 = 0 \rightarrow V_0 = \]
\[ V_2 = V_{CC} \rightarrow V_0 = V_{1A} + V_{1B} \]

---

**Superposition**

\[ V_1 = V_{CC} + 0 \rightarrow V_0 = V_{1A} + V_{1B} \]
\[ V_2 = 0 + V_{CC} \rightarrow V_0 = V_{1A} + V_{1B} \] short circuit

\[ i = -I_0 = 0 \rightarrow \] open circuit
Demo: Jello

Voltage divider

From last Thu Lecture
Norton

- MIT EE grad (1922)

“The illustrative example considered above gives the solution for the ratio of the input to output current, since this seems to be of more practical interest. An electric network usually requires the solution for the case of a constant voltage in series with an output impedance connected to the input of the network. This condition would require the equations of the voltage divided by the current in the load to be treated as above. It is ordinarily easier, however, to make use of a simple theorem which can be easily proved, that the effect of a constant voltage $E$ in series with an impedance $Z$ and the network is the same as a current $I = E/Z$ into a parallel combination of the network and the impedance $Z$. If, as is usually the case, $Z$ is a pure resistance, the solution of this case reduces to the case treated above for the ratio of the two currents, with the additional complication of a resistance shunted across the input terminals of the network. If $Z$ is not a resistance the method still applies, but here the variation of the input current $E/Z$ must be taken into account.”


https://www.ece.rice.edu/~dhj/norton/