Lecture 2

Today

1. Lab 2
2. Nodal analysis
3. Linearity

Steps

1. Ground - DONE
   \[ \frac{V_0}{R_1} = \frac{V_1}{R_2} = \frac{V_2}{R_3} \] and so on
2. Label - DONE
3. KCL @ node \( e_2 \)
   \[ i_1 + i_2 + i_3 = 0 \leftarrow \text{skip} \]
   \[ G_0 (V_0 - e_2) + G_2 (0 - e_2) + G_3 (e_1 - e_2) + I_0 = 0 \]
   \[ \frac{V_0 - e_1}{R_1} + \frac{0 - e_1}{R_2} + \frac{e_2 - e_1}{R_3} = 0 \]

4. Solve: Two ways
   a) Substitute and solve
   b) Use matrix algebra

\[ \begin{pmatrix} G_1 + G_2 + G_3 & -G_3 \\ -G_3 & G_3 + G_4 + G_5 \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} G_1 V_0 \\ I_0 + G_4 V_0 \end{pmatrix} \]

\[ \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{1}{\text{det}(A)} \begin{pmatrix} G_3 + G_4 + G_5 & G_3 \\ G_3 + G_4 + G_5 & G_4 \end{pmatrix} \begin{pmatrix} G_1 V_0 \\ I_0 + G_4 V_0 \end{pmatrix} \]

\[ \text{det}(A) = (G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2 \]

V:\ \[ i_3 = \frac{e_2 - e_1}{R_3} \]
9/16/19

- Topology gives $G + S$
  \[ \Rightarrow \text{can go directly fromckt to matrix} \]
  \[ \downarrow \]
  \[ \text{this is how ckt solves work} \]

$G_{ii} = \sum G_i$'s connected to node i

$G_{ij} = -\sum G_i$'s connected b/t nodes i & j

$S_i = \sum \text{independent current sources into node } i$

\[ \text{+ independent voltage sources \& conductance linking them to node } i \]

3. Linearity

\[ e = G' \cdot S' \]

\[ \text{outputs} \uparrow \quad \text{inputs} \]

From our circuit:

\[ e_i = k_1 V_0 + k_2 I_0 \quad \text{only depend on conductances} \]

\[ e_a = k_3 V_0 + k_4 I_0 \quad \Rightarrow \text{same for any branch} \]

\[ \text{Let's are linear combos \& independent source elements} \]

True for any circuit comprised of constant resistors \& independent sources

\[ \downarrow \]

Linear

Homogeneity

Additivity \Rightarrow \text{Superposition}
Recap

1. Series/parallel relations

![Image of series/parallel circuit]

\[ R_S = R_1 + R_2 + \cdots + R_N \]

2. Voltage and current dividers

![Image of voltage and current divider circuit]

\[ v_2 = \frac{R_2}{R_1 + R_2} V \]

\[ i_2 = \frac{R_1}{R_1 + R_2} I \]

3. Intuitive method

- Collapse circuit using series/parallel and dividers till it is trivial to solve
- Expand circuit back out to solve for branch variable(s) of interest

\[ R_P = \frac{\frac{R_1 R_2}{R_1 + R_2}} = R_1 || R_2 \]

\[ \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \]

\[ G_N = \frac{1}{R_N} \]

Adapted from Lang and Agarwal

6.002

Transducers

Analog electronics

Discretized
Discrete time
aka digital

Continuous-valued
Continuous time
aka analog
DAC: digital-to-analog converter

- Used to create analog voltages from digital computers
  - Play music on a speaker
  - Control rotational velocity of a motor
  - Etc.

How to make?
- PWM + external circuit
- Internal to microcontroller (true DAC)
- Using resistive network → Lab 2
  - Need to understand how to analyze circuits with multiple sources
Nodal analysis

1. **Select a reference node**, called ground, from which all other voltages will be measured. Define its potential to be 0 V.

2. **Label the potentials of the remaining nodes** with respect to the ground node. Any node connected to the ground node through an independent voltage source should be labeled with the voltage of that source. (Also for dependent sources).

3. **Write KCL for each node** that has an unknown node voltage, *immediately* substituting in element law.

4. **Solve the equations** resulting from Step 3 for the unknown node voltages.

5. **Back-solve for the branch voltages and currents**.

\[
\begin{bmatrix}
  e_1 \\
  e_2
\end{bmatrix}
= \frac{1}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} \begin{bmatrix}
  G_3 + G_4 + G_5 \\
  G_3
\end{bmatrix}
\begin{bmatrix}
  G_3 \\
  G_1 + G_2 + G_3
\end{bmatrix}
\begin{bmatrix}
  G_1 V_0 \\
  G_4 V_0 + I_0
\end{bmatrix}
\]

\[
e_1 = \frac{(G_3 + G_4 + G_5)G_1}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} V_0 + \frac{G_3}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} (G_4 V_0 + I_0)
\]

\[
e_1 = \frac{(G_3 + G_4 + G_5)G_1 + G_3 G_4}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} V_0 + \frac{G_3}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} I_0
\]

\[
e_2 = \frac{G_3 G_1}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} V_0 + \frac{G_1 + G_2 + G_3}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} (G_4 V_0 + I_0)
\]

\[
e_2 = \frac{(G_1 + G_2 + G_3)G_4 + G_3 G_1}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} V_0 + \frac{G_1 + G_2 + G_3}{(G_1 + G_2 + G_3)(G_3 + G_4 + G_5) - G_3^2} I_0
\]

\[
e_1 = k_1 V_0 + k_2 I_0
\]

\[
e_2 = k_3 V_0 + k_4 I_0
\]
Linearity

**Homogeneity**

**IF:**

\[ x_1 \rightarrow y_1 \]
\[ \vdots \]
\[ x_N \rightarrow y_N \]

**THEN:**

\[ \alpha x_1 \rightarrow \alpha y_1 \]
\[ \vdots \]
\[ \alpha x_N \rightarrow \alpha y_N \]

If we scale the inputs, the output scales → Zero input → zero output

**Additivity**

**IF:**

\[ x_{1A} \rightarrow y_{1A} \]
\[ \vdots \]
\[ x_{NA} \rightarrow y_{1B} \]

**THEN:**

\[ x_{1A} + x_{1B} \rightarrow y_{1A} + y_{1B} \]
\[ \vdots \]
\[ x_{N1} + x_{NB} \rightarrow y_{1B} \]

Sum of the inputs → sum of responses to individual inputs